



Foundations of Numerical Reasoning in Grades 3-5

Module 2: Building Fluency Through 1000 (Add & Sub)

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6 Week Module Breakdown

Week 1: Whole Number and Place Value

Week 2: Building Fluency through 1000 (Add & Sub)

Week 3: Building Fluency through 100 (Mult & Div)

Week 4: Building Fluency through 1000 (Mult & Div)

Week 5: Extending Place Value and Operations (Decimals)

Week 6: Number Statistical Reasoning

Module Breakdown

Part 1: Launch the Concept

Part 2: Connect and Extend Thinking

Week 3: Building Fluency



3-Act Task

The Banana Bunch



Act 1



Notice?

Pause
II

Wonder?



How much will the 2 bananas weigh?

Pause
II



Act 2

What information do you need?

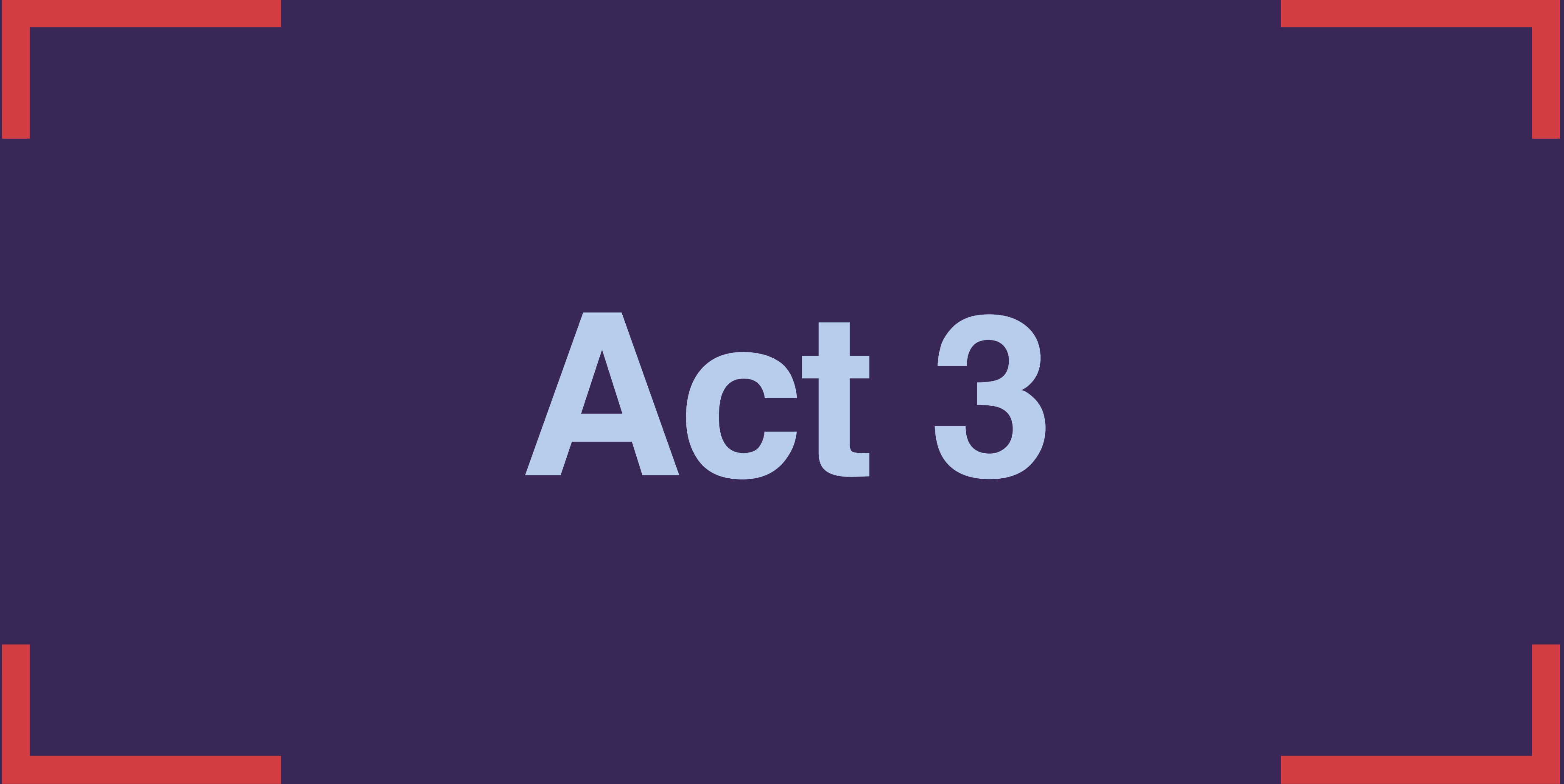
Pause
II



Pause
II



The banana weighs 156 grams



Act 3





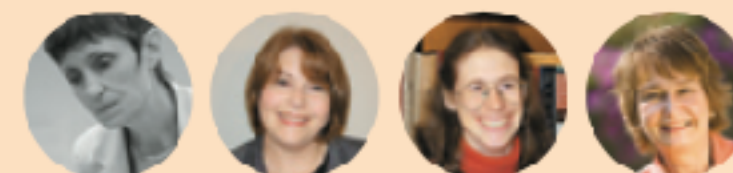
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Orchestrating Discussions

Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein



Margaret S. Smith, pegs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. Elizabeth K. Hughes, elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. Randi A. Engle, raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. Mary Kay Stein, mstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important

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The 5 Practices

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.



The 5 Practices

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
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4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

Learning Target:

Questions and Look-Fors:

Strategy	Who and What	Order
		

Notes:

Anticipate

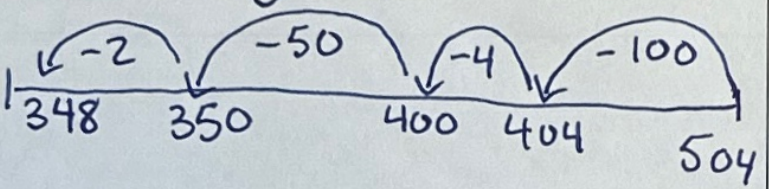
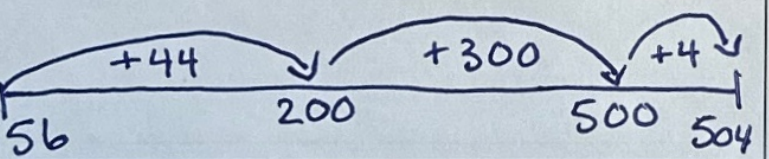


Pause
II

Task Planning Page

Learning Target: Subtraction within 1000

Questions and Look-Fors: Thinking addition
Place value strategies - counting backwards/forwards 100,10,1

Strategy	Who and What	Order
Counting back 		
Thinking addition (adding up) 		
$\begin{array}{r} 400 \overset{90}{100} \\ 500 + 0 + 14 \\ - 100 + 50 + 6 \\ \hline 300 + 40 + 8 \end{array}$		
Constant Difference $504 - 156 = 500 - 152$ $(500 - 150) - 2$ $350 - 2 = 348$		
Standard algorithm $\begin{array}{r} 4\overset{9}{\cancel{5}}04 \\ 156 \\ \hline 248 \end{array}$		
Notes:		

Anticipating → Monitoring → Selecting → Sequencing → Connecting

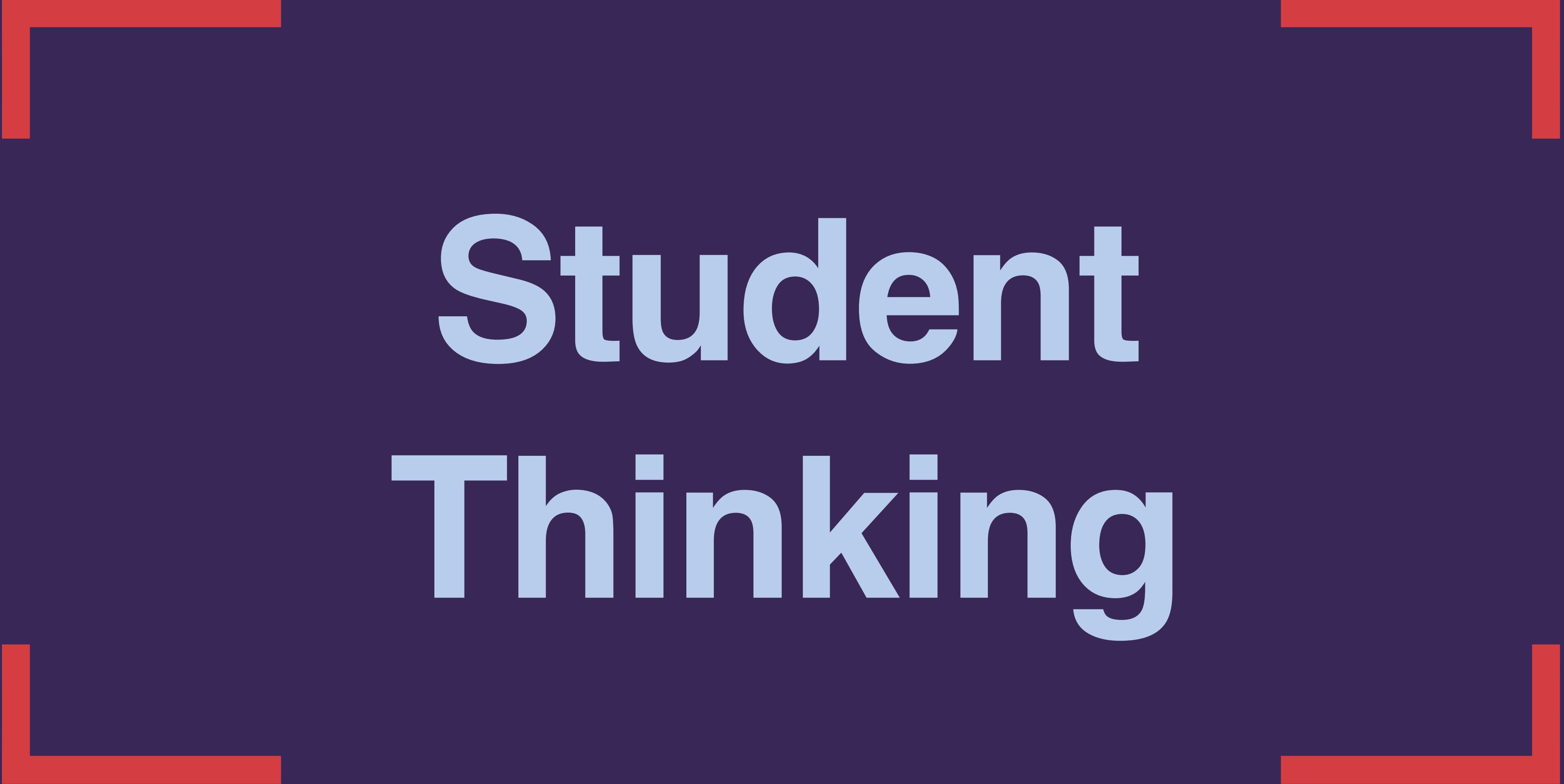
Anticipate





The 5 Practices

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.



Student Thinking

1. What did you notice? banana 504	2. What do you wonder? How much does each banana weigh?
3. Main Question: How much does one banana weigh?	
4. Estimate: 176	5. What information do you need? How much do the 2 bananas weigh?
6. Show your thinking: $\begin{array}{r} 156 + 150 = 306 \\ 160 + 140 = 300 \\ 200 + 100 = 300 \\ 504 - 150 = 354 \\ 504 - 160 = 344 \\ 504 - 200 = 304 \\ 504 - 150 + 4 = 354 \\ 504 - 160 + 4 = 344 \\ 504 - 200 + 4 = 304 \end{array}$	

STUDENT 1

1. What did you notice? A scale	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bananas weigh?	
4. Estimate: 168	5. What information do you need? Weight of 1 banana
6. Show your thinking: $\begin{array}{r} 504 \\ 150 \quad 150 \quad 150 \\ 15 \quad 15 \quad 15 \\ 3 \quad 3 \quad 3 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ - 150 \\ \hline 354 \end{array}$	

STUDENT 2

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1 banana weigh?	
4. Estimate: 150	5. What information do you need? What does 2 bananas weigh?
6. Show your thinking: $\begin{array}{r} 4 \times 14 \\ 8 \times 4 \\ - 156 \\ \hline 348 \end{array}$	

STUDENT 3

1. What did you notice? Man with heavy arms	2. What do you wonder? Weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? Weight of 2 bananas
6. Show your thinking: $\begin{array}{r} 400 \quad 100 \quad 14 \\ 504 = 500 + 0 + 4 \\ 156 = 100 \quad 50 + 6 \\ \hline 300 + 40 + 8 \\ \hline 348 \end{array}$	

STUDENT 4

Pause
II

1. What did you notice? I notice that put the bunch on the scale and it weighed 504g.	2. What do you wonder? I wonder if we're hanging each banana.
3. Main Question: How heavy are two bananas?	
4. Estimate: 454	5. What information do you need? I need to know how much two bananas weigh.
6. Show your thinking: $\begin{array}{r} 504 \\ - 156 \\ \hline 348 \end{array}$	

STUDENT 5

1. What did you notice? On the scale and weighed 504g. Separated 2 bananas.	2. What do you wonder? Why did he take off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed?	
4. Estimate: 375	5. What information do you need? How much does 1 banana weigh?
6. Show your thinking: $\begin{array}{r} 504 \\ - 156 \\ \hline 348 \end{array}$	

STUDENT 6

1. What did you notice? I noticed that it weighed 504 grams.	2. What do you wonder? I wonder if it can be split into three parts?
3. Main Question: How much do the two bananas weigh after the one is removed?	
4. Estimate: 312 grams	5. What information do you need? The information we need is how much the one banana weighs.
6. Show your thinking: $\begin{array}{l} 500 - 100 = 400 \\ 400 - 50 = 350 \\ 350 - 15 = 335 \\ 335 + 13 = 348 \end{array}$	

STUDENT 7

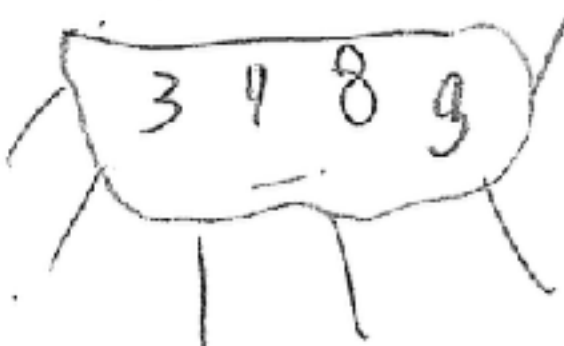
1. What did you notice? 1.5 bananas = 504 grams	2. What do you wonder? 1. How much do 2 bananas weigh? 2. How much does each banana weigh?
3. Main Question: How much does each banana weigh?	
4. Estimate: 180 grams	5. What information do you need? How much do the other two bananas weigh?
6. Show your thinking: $\begin{array}{r} 504 \text{ grams} \\ 44 + 300 + 4 = 348 \text{ grams} \end{array}$	

STUDENT 8

1. What did you notice? banned 504	2. What do you wonder? How much does each banana weigh.
3. Main Question: how much does one banana weigh	
4. Estimate: 176	5. What information do you need? how much do the 2 bananas weigh.
6. Show your thinking: $ \begin{array}{r} 156 + 4 = 160 \\ 160 + 40 = 200 \\ 200 + 300 = 500 \\ 500 + 4 = 504 \\ 200 + 40 = 240 \\ 4 + 4 = 8 \\ 240 + 8 = 248 \end{array} $	

STUDENT 1

1. What did you notice? As call 3 Bannanas	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bannanas weigh?	
4. Estimate: 168	5. What information do you need? Weigh + of 1 bannanna
6. Show your thinking: $\begin{array}{r} 504 \\ 150 \quad 150 \quad 150 \\ 15 \quad 15 \quad 15 \\ 3 \quad 3 \quad 3 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ - 150 \\ \hline 354 \end{array}$ 348	

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1. 150g ???? banana weigh	
4. Estimate: ↓	5. What information do you need? What does 2 bananas weigh
6. Show your thinking: $\begin{array}{r} 4 \cancel{1} 14 \\ 5 \cancel{0} 4 \\ \sim 156 \end{array}$ 	

STUDENT 3

1. What did you notice? Man with hairy arms 11×3	2. What do you wonder? weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? weight of 2 bananas
6. Show your thinking: $504 = 500 + 0 + 4$ $156 = 100 + 50 + 6$ $\begin{array}{r} 300 + 40 + 8 \\ \hline 348 \end{array}$	

1. What did you notice? I notice that put the bunch on the scale and took one off.	2. What do you wonder? I wonder if we're wanging each banana.
3. Main Question: How heavy are two bananas?	
4. Estimate: 454 grams	5. What information do you need? I need to know how much two bananas weigh.
6. Show your thinking: $509 - 156 = 353$ $\swarrow \quad \searrow$ 100 50 6 $509 - 100 = 409$ $409 - 50 = 359$ $359 - 6 = 353$ boo! $\downarrow$ 312	

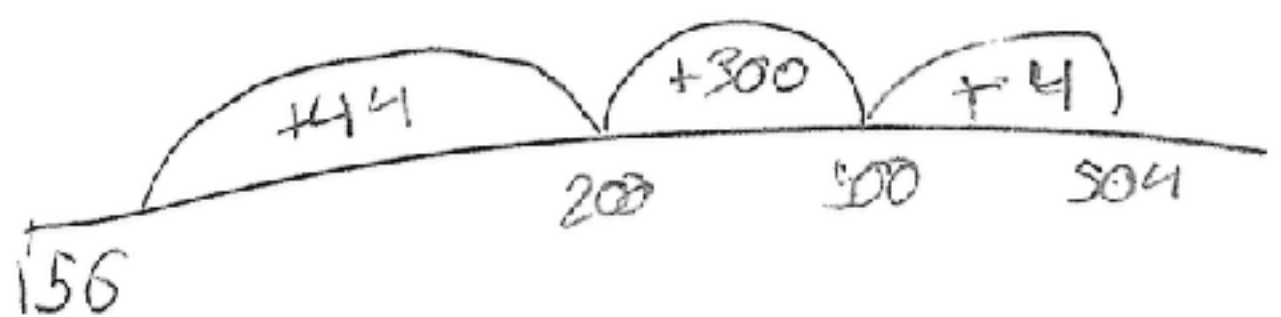
STUDENT 5

1. What did you notice? on the scale and weighed 504g. Separated 1 banana	2. What do you wonder? why did he took off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed.	
4. Estimate: 375g	5. What information do you need? How much does 1 banana weigh!
6. Show your thinking: $\begin{array}{r} 509 \\ -156 \\ \hline 353 \end{array}$	

STUDENT 6

1. What did you notice? I noticed that it weighed 509 grams.	2. What do you wonder? I wonder if it can be split into three parts? equal
3. Main Question: How much do the two bananas weigh after the one is removed	
4. Estimate: 312 grams	5. What information do you need? The information we need is how much the one banana weighs
6. Show your thinking: $500 - 100 = 400$ $400 - 50 = 350$ $350 - 5 = 345$ $350 + 3 = 353$ The answer is 353	

STUDENT 7

1. What did you notice? 1. 3 bananas = 5 24 grams 2. scale	2. What do you wonder? why 3 bananas 1. how much do 2 bananas weigh 2. how much does each weigh
3. Main Question: How much does each banana weigh	
4. Estimate: 100 grams	5. What information do you need? how much do the other two bananas weigh?
6. Show your thinking: 504 grams $44 + 300 + 4 = 348 \text{ grams}$  the two other bananas weigh 348 grams	

STUDENT 8



The 5 Practices

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
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4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

1. What did you notice? banana 504	2. What do you wonder? How much does each banana weigh?
3. Main Question: How much does one banana weigh?	
4. Estimate: 176	5. What information do you need? how much do the 2 bananas weigh.
6. Show your thinking: $ \begin{array}{r} 156 + 17 = 160 \\ 160 + 140 = 200 \\ 200 + 300 = 500 \\ 500 + 4 = 504 \\ 200 + 400 = 600 \\ 4 + 4 = 8 \\ 140 + 8 = 148 \end{array} $	

STUDENT 1

1. What did you notice? A scale	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bananas weigh?	
4. Estimate: 168	5. What information do you need? weigh + of 1 banana
6. Show your thinking: $\begin{array}{r} 504 \\ 150 \quad 150 \quad 150 \\ 15 \quad 15 \quad 15 \\ 3 \quad 3 \quad 3 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ - 150 \\ \hline 354 \end{array}$	

STUDENT 2

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1 banana weigh	
4. Estimate: 150g	5. What information do you need? What does 2 bananas weigh?
6. Show your thinking: $ \begin{array}{r} 4 \cancel{0} 14 \\ 8 \cancel{0} 8 \\ - 156 \\ \hline 348g \end{array} $	

STUDENT 3

1. What did you notice? Man with hairy arms	2. What do you wonder? weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? weight of 2 bananas
6. Show your thinking: $ \begin{array}{r} 400 \quad 100 \quad 14 \\ 504 = 500 + 0 + 4 \\ 156 = 100 \quad 50 + 6 \\ \hline 300 + 40 + 8 \\ \hline 348 \end{array} $	

STUDENT 4

Pause

II

1. What did you notice? I notice that put the bunch on the scale and took one off.	2. What do you wonder? I wonder if we're hanging each banana.
3. Main Question: How heavy are two bananas?	
4. Estimate: 454 grams	5. What information do you need? I need to know how much two bananas weigh.
6. Show your thinking: $ \begin{array}{r} 509 - 156 = 353 \end{array} $ $\begin{array}{r} 100 \quad 50 \quad 6 \\ 509 - 100 = 409 \\ 409 - 50 = 359 \\ 359 - 6 = 353 \end{array}$	

STUDENT 5

1. What did you notice? on the scale and weighed 504g. Separated 1 banana	2. What do you wonder? why did he took off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed?	
4. Estimate: 375g	5. What information do you need? How much does 1 banana weigh?
6. Show your thinking: $ \begin{array}{r} 504 \\ - 156 \\ \hline 358 \end{array} $	

STUDENT 6

1. What did you notice? I noticed that it weighed 504 grams.	2. What do you wonder? I wonder if it can be split into three parts?
3. Main Question: How much do the two bananas weigh after the one is removed?	
4. Estimate: 312 grams	5. What information do you need? The information we need is how much the one banana weighs
6. Show your thinking: $ \begin{array}{l} 500 - 100 = 400 \\ 400 - 50 = 350 \\ 350 - 6 = 344 \\ 350 + 3 = 353 \end{array} $ The answer is 353	

STUDENT 7

1. What did you notice? 1.5 bananas = 504 grams	2. What do you wonder? 1. how much do 2 bananas weigh? 2. how much does each banana weigh?
3. Main Question: How much does each banana weigh?	
4. Estimate: 100 grams	5. What information do you need? how much do the other two bananas weigh?
6. Show your thinking: $ \begin{array}{r} 504 \text{ grams} \\ 44 + 300 + 4 = 348 \text{ grams} \\ \hline 156 \quad 200 \quad 500 \quad 504 \end{array} $ the two other bananas weigh 348 grams	

STUDENT 8

1. What did you notice? banana on the scale and weighed 504g. Separated 1 banana	2. What do you wonder? why did he took off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed.	
4. Estimate: 375g	5. What information do you need? How much does 1 banana weigh?
6. Show your thinking: $\begin{array}{r} 504 \\ -156 \\ \hline 348 \end{array}$	

STUDENT 6

1. What did you notice? I noticed that it weighed 504 grams.	2. What do you wonder? I wonder if it can be split into three parts? equal
3. Main Question: How much do the two bananas weigh after the one is removed	
4. Estimate: 312 grams	5. What information do you need? The information we need is how much the one banana weighs
6. Show your thinking: $500 - 100 = 400$ $400 - 50 = 350$ $350 - 6 = 344$ $350 + 3 = 353$ the answer is 353	

STUDENT 7

1. What did you notice? I notice that put the bunch on the scale and took one off.	2. What do you wonder? I wonder if we're hanging each banana.
3. Main Question: How heavy are two bananas?	
4. Estimate: 454 grams	5. What information do you need? I need to know how much two bananas weigh.
6. Show your thinking: $509 - 156 = 353$ 100 50 6 $509 - 100 = 409$ $409 - 50 = 359$ $359 - 6 = 353$ boo!	

STUDENT 5

1. What did you notice? A scale 3 Bananas	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bananas weigh?	
4. Estimate: 168	5. What information do you need? Weigh + of 1 banana
6. Show your thinking: $\begin{array}{r} 504 \\ -150 \\ \hline 354 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ -150 \\ \hline 354 \end{array}$ 348	

STUDENT 2

1. What did you notice? 1.5 bananas = 504 grams 2 scale	2. What do you wonder? 1. how much do 2 bananas weigh? 2. how much does each weigh?
3. Main Question: How much does each banana weigh?	
4. Estimate: 100 grams	5. What information do you need? how much do the other two bananas weigh?
6. Show your thinking: 504 grams $44 + 300 + 4 = 348 \text{ grams}$ the two other bananas weigh 348 grams	

STUDENT 8

1. What did you notice? banana weigh 504	2. What do you wonder? How much does each banana weigh.
3. Main Question: how much does one weigh?	
4. Estimate: 176	5. What information do you need? how much do the 2 bananas weigh.
6. Show your thinking: $156 + 14 = 170$ $170 + 40 = 210$ $210 + 300 = 510$ $510 - 10 = 500$ $500 + 4 = 504$ $4 + 4 = 8$ $170 + 8 = 178$	

STUDENT 1

1. What did you notice? Man with hairy arms x3	2. What do you wonder? weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? weight of 2 bananas
6. Show your thinking: $504 = 500 + 4$ $156 = 100 + 50 + 6$ $300 + 40 + 8 = 348$	

STUDENT 4

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1 banana weigh	
4. Estimate: 150g	5. What information do you need? what does 2 bananas weigh?
6. Show your thinking: $\begin{array}{r} 4 \text{ } 14 \\ 504 \\ -156 \\ \hline 348 \end{array}$	

STUDENT 3



The 5 Practices

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students' work on and engagement with the tasks;
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4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

1. What did you notice? banana on the scale and weighed 504g. Separated 1 banana	2. What do you wonder? why did he took off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed.	
4. Estimate: 375g	5. What information do you need? How much does 1 banana weigh?
6. Show your thinking: $\begin{array}{r} 504 \\ -156 \\ \hline 348 \end{array}$	

STUDENT 6

1. What did you notice? I noticed that it weighed 504 grams.	2. What do you wonder? I wonder if it can be split into three parts? equal
3. Main Question: How much do the two bananas weigh after the one is removed	
4. Estimate: 312 grams	5. What information do you need? The information we need is how much the one banana weighs
6. Show your thinking: $500 - 100 = 400$ $400 - 50 = 350$ $350 - 10 = 340$ $340 + 13 = 353$ the answer is 353	

STUDENT 7

1. What did you notice? I notice that put the bunch on the scale and took one off.	2. What do you wonder? I wonder if we're weighing each banana.
3. Main Question: How heavy are two bananas?	
4. Estimate: 454 grams	5. What information do you need? I need to know how much two bananas weigh.
6. Show your thinking: $509 - 156 = 353$ 100 50 6 $509 - 100 = 409$ $409 - 50 = 359$ $359 - 6 = 353$ 353	

STUDENT 5

1. What did you notice? A scale 3 Bananas	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bananas weigh?	
4. Estimate: 168	5. What information do you need? Weigh 1 banana
6. Show your thinking: $\begin{array}{r} 504 \\ -150 \\ \hline 354 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ -150 \\ \hline 354 \end{array}$	

STUDENT 2

Pause
II

1. What did you notice? 1.5 bananas = 504 grams 2 scale	2. What do you wonder? 1. how much do 2 bananas weigh? 2. how much does each weigh?
3. Main Question: How much does each banana weigh?	
4. Estimate: 100 grams	5. What information do you need? how much do the other two bananas weigh?
6. Show your thinking: 504 grams $44 + 300 + 4 = 348 \text{ grams}$ the two other bananas weigh 348 grams	

STUDENT 8

1. What did you notice? banana weigh 504	2. What do you wonder? How much does each banana weigh?
3. Main Question: how much does one banana weigh?	
4. Estimate: 176	5. What information do you need? how much do the 2 bananas weigh.
6. Show your thinking: $156 + 14 = 170$ $170 + 40 = 210$ $210 + 300 = 510$ $510 - 10 = 500$ $500 + 4 = 504$	

STUDENT 1

1. What did you notice? Man with hairy arms 1x3	2. What do you wonder? weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? weight of 2 bananas
6. Show your thinking: $504 = 500 + 4$ $156 = 100 + 50 + 6$ $300 + 40 + 8 = 348$	

STUDENT 4

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1 banana weigh	
4. Estimate: 150g	5. What information do you need? What does 2 bananas weigh?
6. Show your thinking: $\begin{array}{r} 4 \text{ } 14 \\ 504 \\ -156 \\ \hline 348 \end{array}$	

STUDENT 3

1. What did you notice? A scale 3 Bananas	2. What do you wonder? Is that pounds
3. Main Question: How much do the 2 bananas weigh?	
4. Estimate: 168	5. What information do you need? Weigh + of 1 banana
6. Show your thinking: $\begin{array}{r} 504 \\ \swarrow \quad \downarrow \quad \searrow \\ 150 \quad 150 \quad 150 \\ 15 \quad \quad 15 \quad 15 \\ 3 \quad \quad \quad 3 \quad 3 \end{array}$ Step 1 Step 2 $\begin{array}{r} 504 \\ - 150 \\ \hline 354 \end{array}$ 348	

STUDENT 2

1. What did you notice? 1.5 bananas = 504 grams 2 scale	2. What do you wonder? 1. how much do 2 bananas weigh? 2. how much does each weigh?
3. Main Question: How much does each banana weigh?	
4. Estimate: 100 grams	5. What information do you need? how much do the other two bananas weigh?
6. Show your thinking: 504 grams $44 + 300 + 4 = 348 \text{ grams}$ the two other bananas weigh 348 grams	

STUDENT 8

1. What did you notice? banana 504	2. What do you wonder? How much does each banana weigh?
3. Main Question: how much does one banana weigh?	
4. Estimate: 176	5. What information do you need? how much do the 2 bananas weigh?
6. Show your thinking: $\begin{array}{l} 156 + 14 = 160 \\ 160 + 140 = 200 \\ 200 + 300 = 500 \\ 500 + 4 = 504 \\ 200 + 400 = 600 \\ 4 + 4 = 8 \\ 140 + 8 = 148 \\ 148 + 200 = 348 \end{array}$	

STUDENT 1



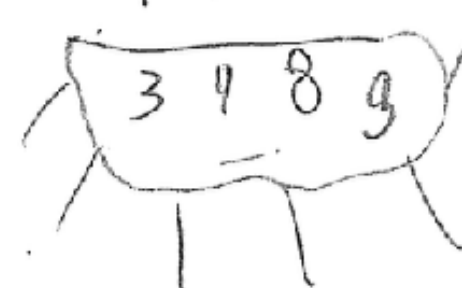
Making Different Connections

1. What did you notice? on the scale and banana weighed 504g. Separated 1 banana	2. What do you wonder? why did he took off one of the bananas?
3. Main Question: How much do the two bananas weigh after one is removed.	
4. Estimate: 375g	5. What information do you need? How much does 1 banana weigh!
6. Show your thinking: $\begin{array}{r} 504 \\ -156 \\ \hline 348 \end{array}$	

STUDENT 6

1. What did you notice? Man with hairy arms 1×3	2. What do you wonder? weight of banana
3. Main Question: Weight of 2 bananas	
4. Estimate: 150	5. What information do you need? weight of 2 bananas
6. Show your thinking: $\begin{array}{r} 400 \quad 90 \quad 14 \\ 504 = 500 + 0 + 4 \\ 156 = 100 \quad 50 + 6 \\ \hline 300 + 40 + 8 \\ \hline 348 \end{array}$	

STUDENT 4

1. What did you notice? 3 bananas it weighs 504	2. What do you wonder? Why does he have 3 bananas
3. Main Question: How much did the 1 banana weigh 150g ???? banana weigh	
4. Estimate: ↓	5. What information do you need? what does 2 bananas weigh
6. Show your thinking: $\begin{array}{r} 4 \quad 90 \quad 14 \\ 504 \\ -156 \\ \hline 348 \end{array}$ 	

STUDENT 3

Module Breakdown

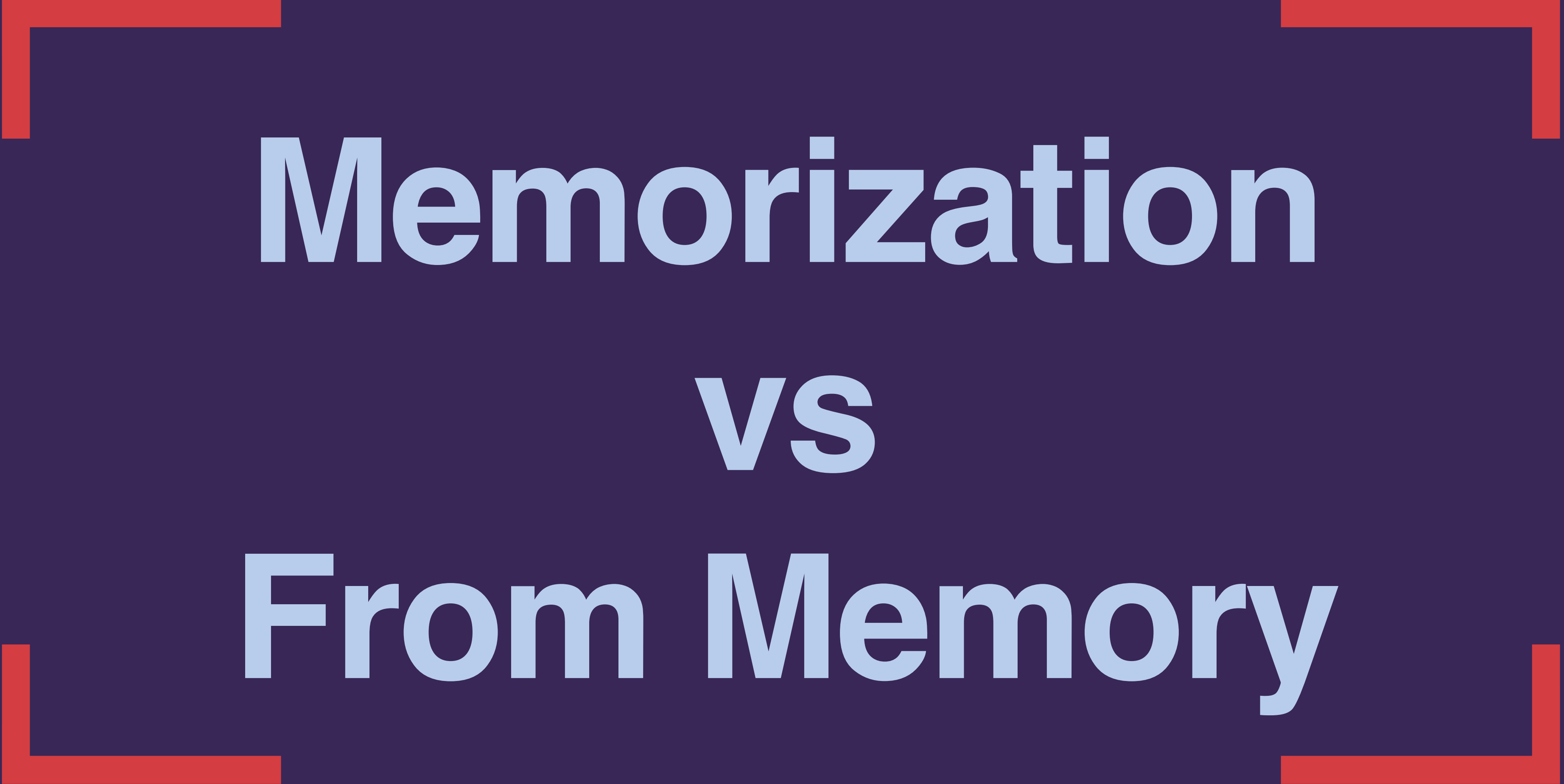
Part 1: Launch the Concept

Part 2: Connect and Extend Thinking

Week 3: Building Fluency

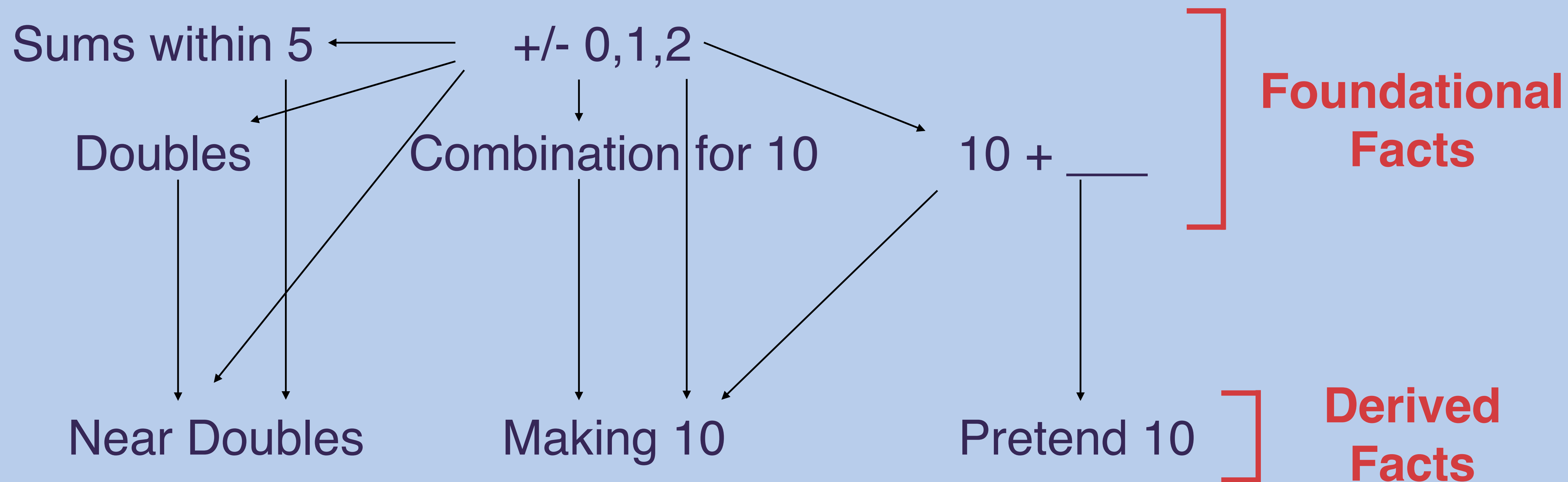


Facts within 20



Memorization vs From Memory

Addition & Subtraction Fact Fluency



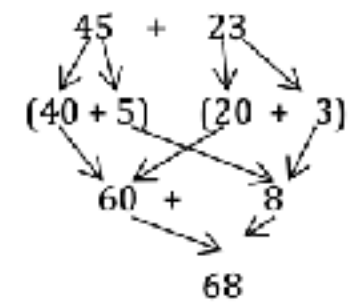
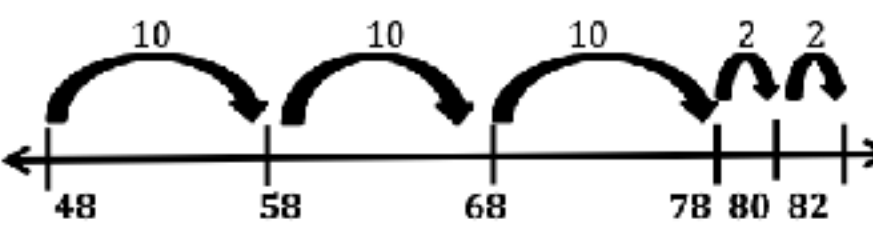
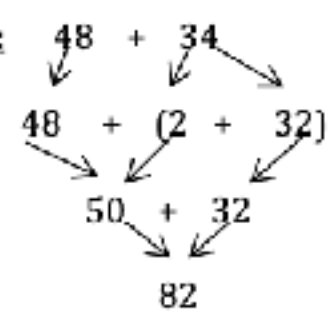
Make 10

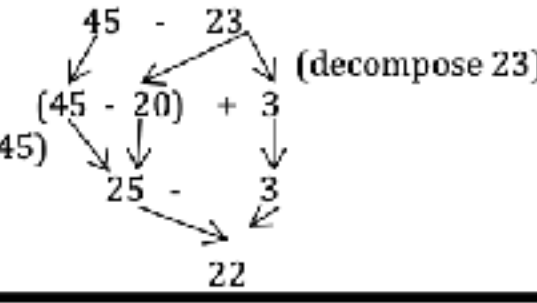
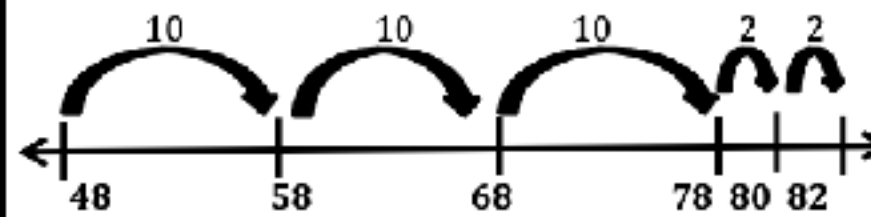
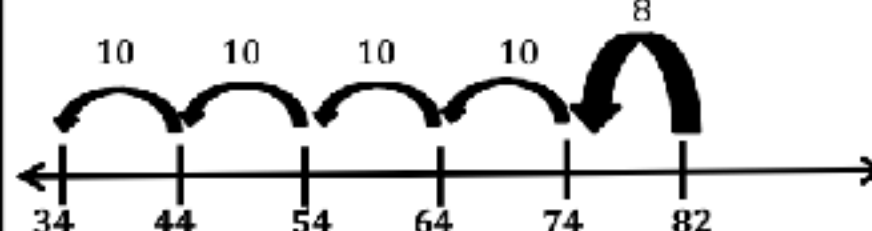
$$9 + 7 = 10 + 6$$

Pretend a 10

$$9 + 7 = (10 + 7) - 1$$

Strategies for Addition and Subtraction

Counting All/Counting On Question: 8+3 Sample Solution: For counting all the students would combine 8 and 3 by counting the set (1, 2, 3, 4, 5, 6, 7, 8....9, 10, 11) For counting on the student could say "8....9, 10, 11"	Breaking Up Into Place Value Question: 45 + 23 Sample Solution: 
Making Tens Question: 9+4 Sample Solution: Student could say "I decomposed the 4 (3 and 1) and gave one to the 9 to make a ten and added the remaining 3." $9+4 = 10+3$	Adding Up In Chunks Question: 48+34 Sample Solution: 48+34 
Doubles/Near Doubles Question: 8+7 (when students use their double facts to solve related problems) Sample Solution: $8+7 = 7+7+1$ $8+7 = 8+8-1$	Compensation Question: 39 + 57 Sample Solution: $39 + 57$ $+1 -1$ $40 + 56 = 96$ Compensation: removing one quantity from one addend and adding it to the other addend. Although quantities are manipulated the total sum remains the same.
Landmark/Friendly Numbers Question: 48+34 Sample Solution: 	Adjusting 1 Number To Create An Easier Number Question: 39 + 24 Sample Solution: Adding one to 39 to make it a 40 $(39 [+1]) + 24$ $(40) + 24$ $64 [-1] = 63$ Added 1 to 39 so 1 was removed from the sum

Counting Back Question: 8-3 Sample Solution: For counting back students would start at 8 and count backward 3 until they arrived at 5. 8....7, 6, 5	Removal in Parts Question: 45 - 23 Sample Solution: 				
Constant Difference Question: 57-22 Sample Solution: Add 3 to each number and the difference remains the same. Only the numbers become friendlier to work with. $57 - 22$ $+3 +3$ (add 3 to each # keeps difference the same) $60 - 25$ $60-25=35$	Adding Up to find the Difference Question: 82-48 Sample Solution: 82-48 $48 + (10 + 10 + 10 + 4) = 82$  Student adds up from 48 to 82 to find the difference of 34.				
Part Whole Box Model Question: 57-22 Sample Solution: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="2">Whole 57</td> </tr> <tr> <td>Part 22</td> <td>Part 35</td> </tr> </table> Students understand the whole and one part of the whole. Because of this, the student is able to identify the other missing part of the whole.	Whole 57		Part 22	Part 35	Adjusting 1 Number To Create An Easier Number Question: 39 - 24 Sample Solution: Adding one to 39 to make it a 40 $(39 [+1]) - 24$ $(40) - 24 = 16$ $16 [-1] = 15$ Added 1 to 39 so 1 was removed from the sum
Whole 57					
Part 22	Part 35				
Using a Number Line Question: 82-48 Sample Solution: 82-48  Student adds up from 48 to 82 to find the difference					

Pause
II

Properties of Addition

Additive Identity Property of Zero

Commutative Property

Associative Property

Additive Identity Property of Zero

$$*a* + 0 = *a*$$

Commutative Property of Addition

$$2 + 9 = 9 + 2$$

Associative Property of Addition

$$9 + 7$$

$$9 + (1 + 6)$$

$$(9 + 1) + 6$$

$$10 + 6$$

Why?

Associative Property of Addition

$$9 + 7$$

$$58 + 7$$

$$248 + 34$$

$$5.8 + 3.4$$



**Take it to
the Bank**



MAKING SENSE SERIES

Progression of Base Ten Models

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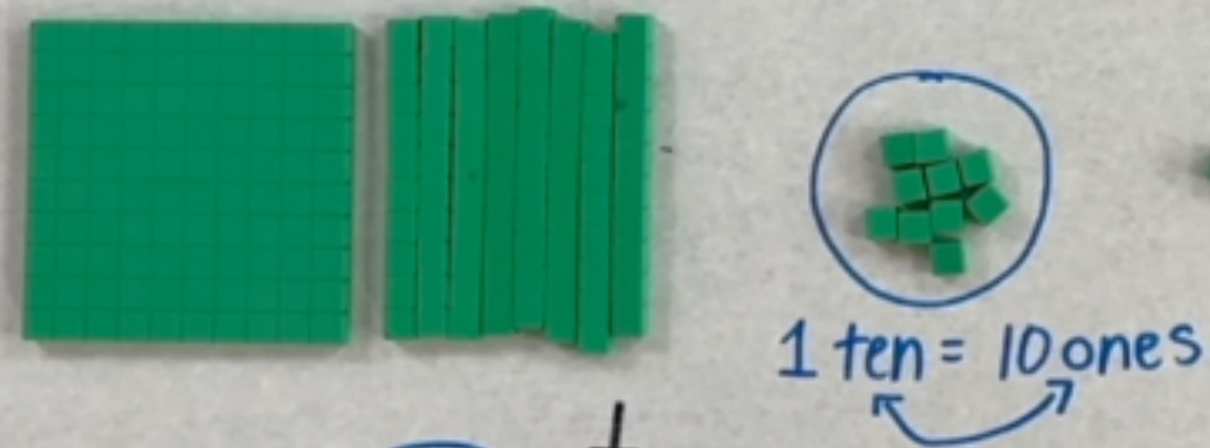
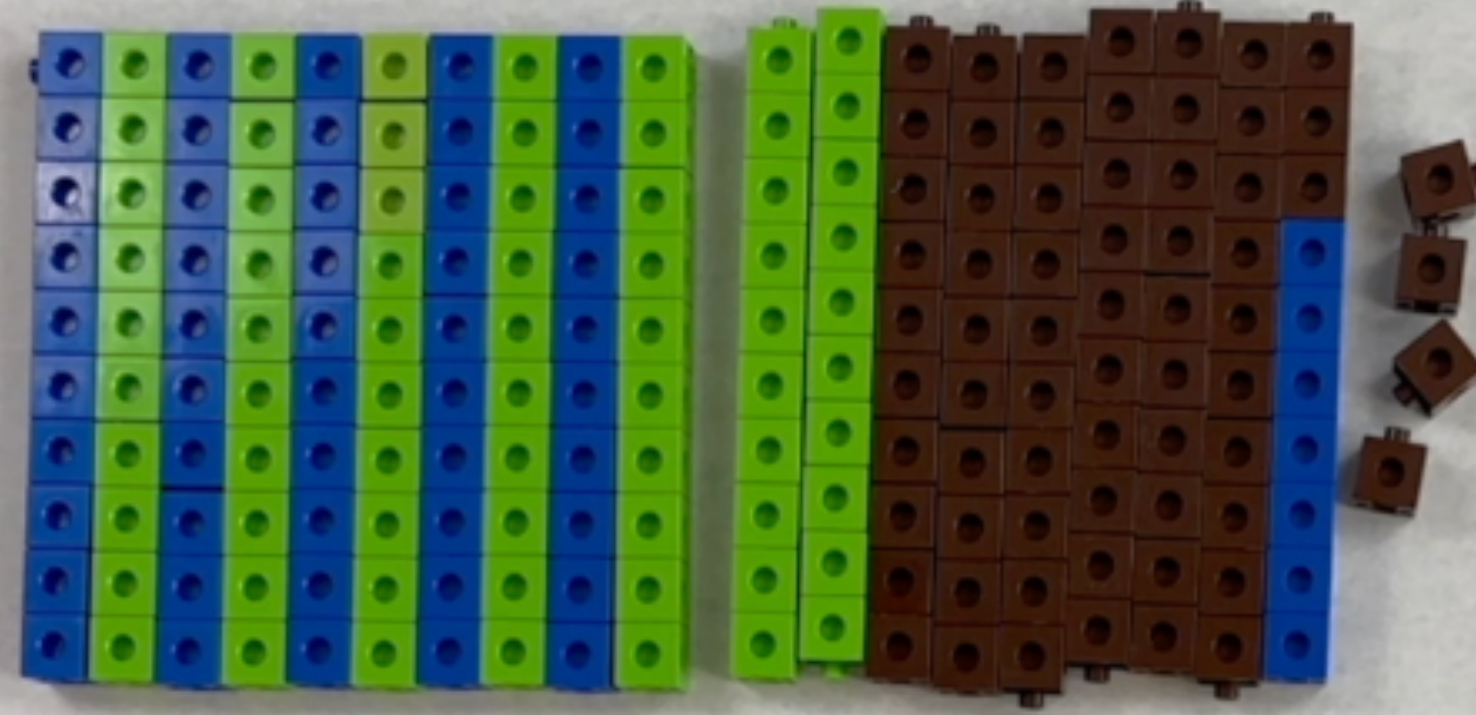
Base Ten Models for Operations

Concrete

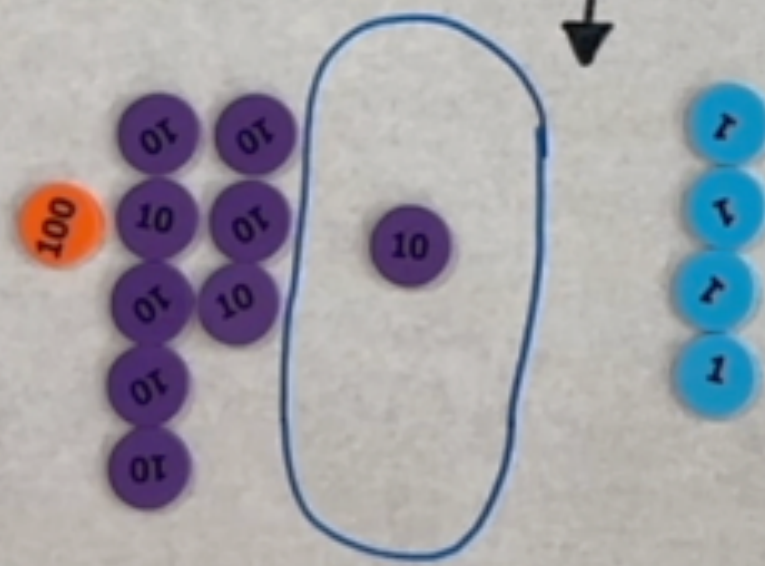
Representational

Abstract

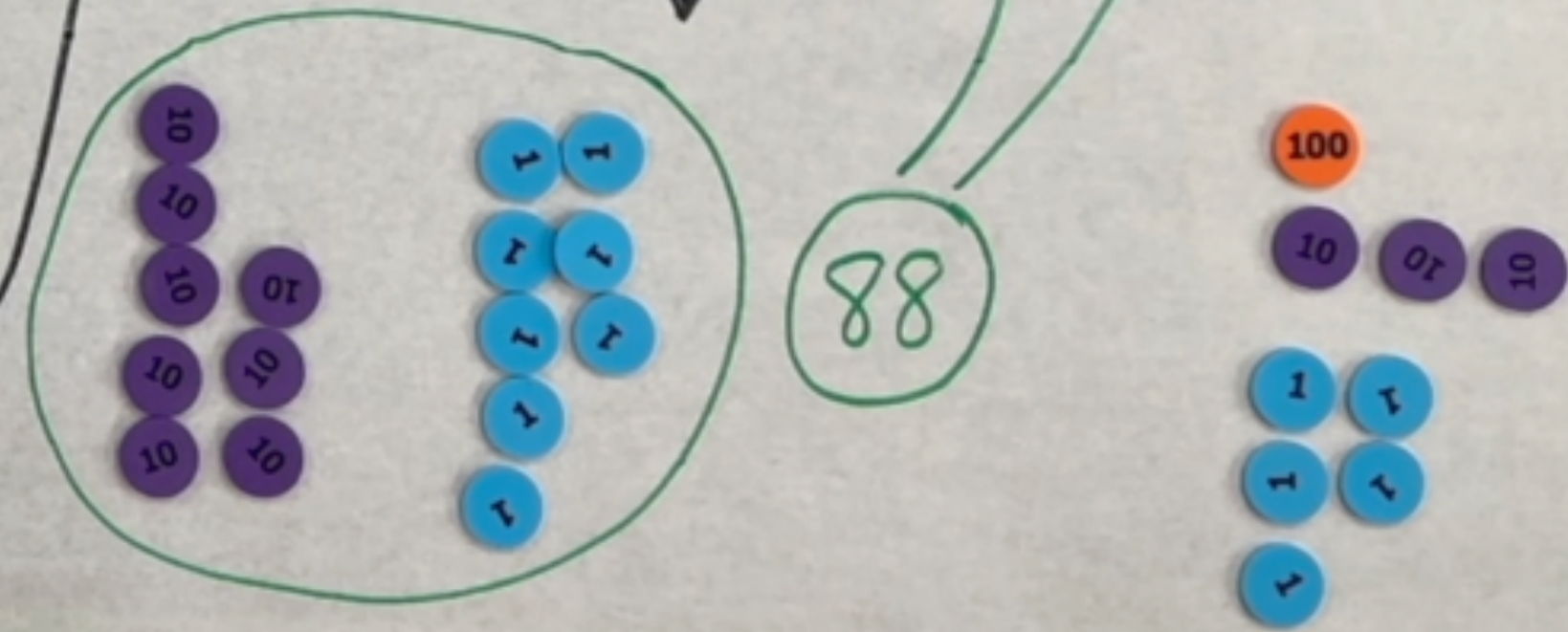
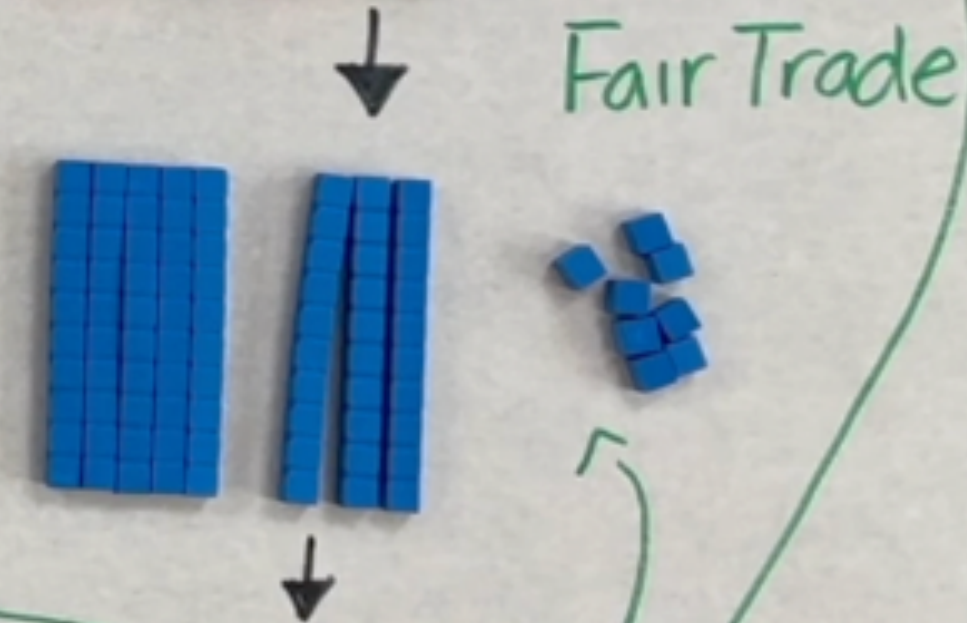
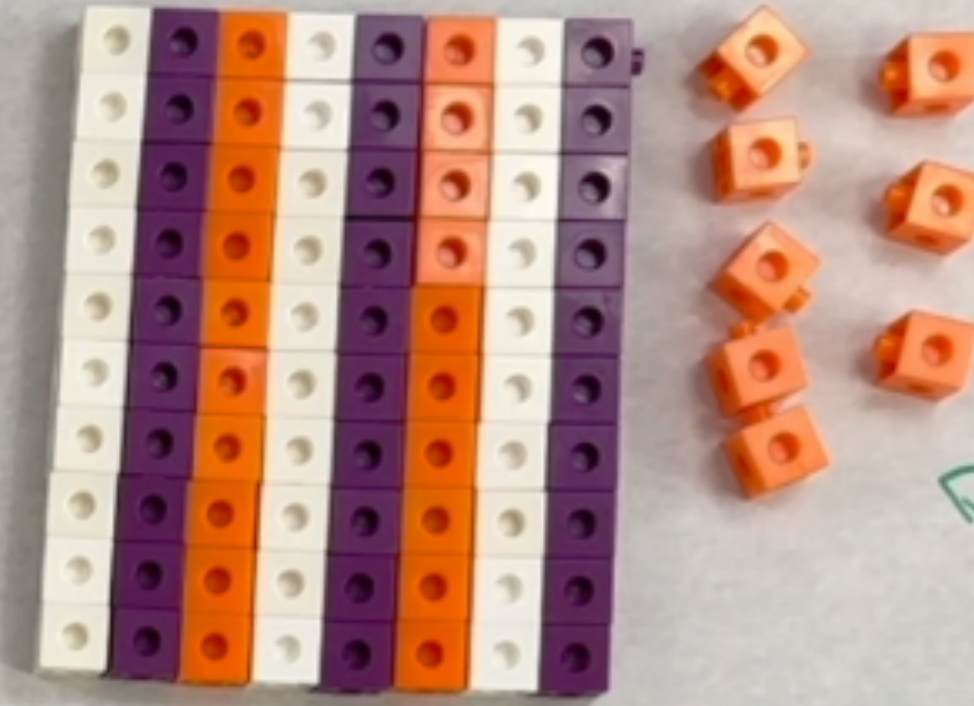
$$127 + 67 = 194$$



100 = 100
10 = 10
1 = 1



$$223 - 135 = 88$$





MAKING SENSE SERIES

Progression of Subtraction with Regrouping

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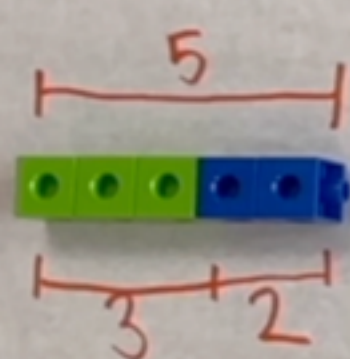
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Subtracting with Regrouping

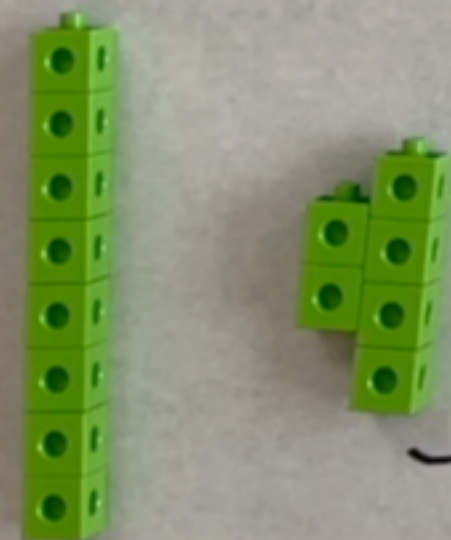
$$5 - 2$$

Take-Away / Separate

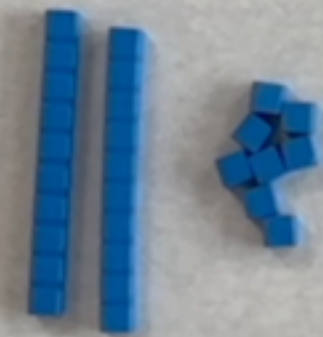


Within 20

$$14 - 6 = 8$$

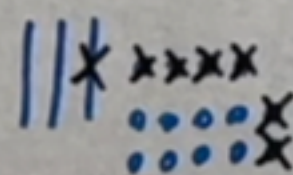


$$34 - 6$$

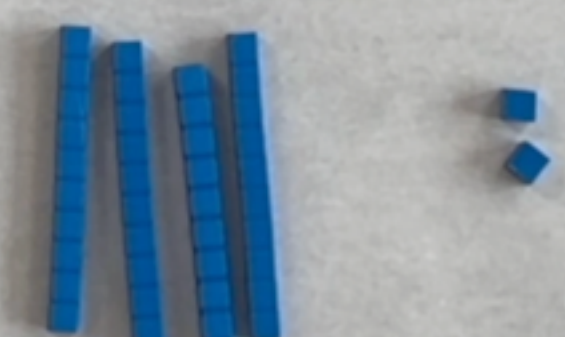
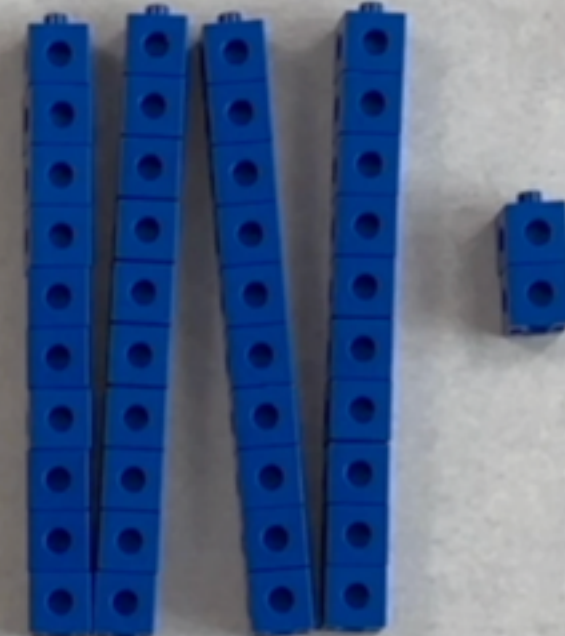


$$\begin{array}{r} 34 \Rightarrow 30 + 4 \\ - 6 \Rightarrow - \quad 6 \\ \hline 20 + 8 \end{array}$$

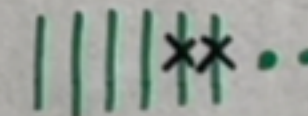
quick draw



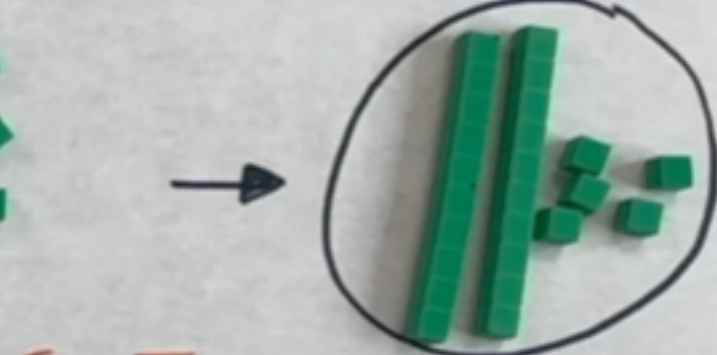
$$62 - 20$$



$$\begin{array}{r} 62 \rightarrow 60 + 2 \\ - 20 \rightarrow 20 + 0 \\ \hline 40 + 2 \end{array}$$



$$63 - 25$$



Fair Trade

$$\begin{array}{r} 63 \rightarrow 50 + 13 \\ - 25 \rightarrow 20 + 5 \\ \hline 30 + 8 \end{array}$$

$$\begin{array}{r} 421 \rightarrow 300 + 120 + 1 \\ - 167 \rightarrow 100 + 60 + 7 \\ \hline 254 \end{array}$$



MAKING SENSE SERIES

Progression of Addition & Subtraction

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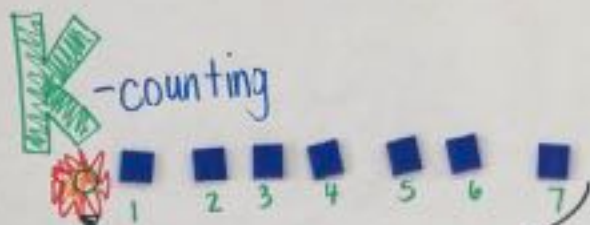
@gfletchy



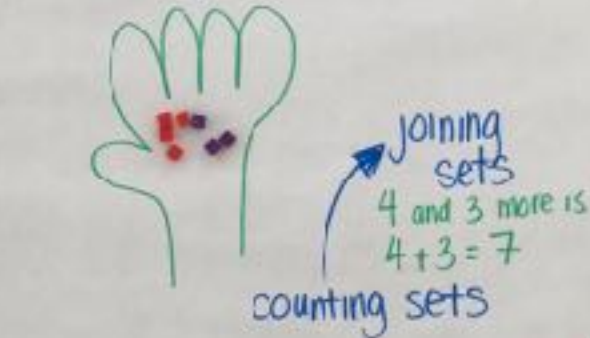
gfletchy@gmail.com



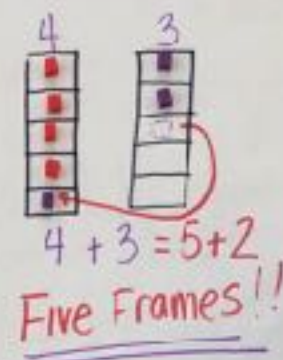
The progression of addition and subtraction



4 and 3



counting sets

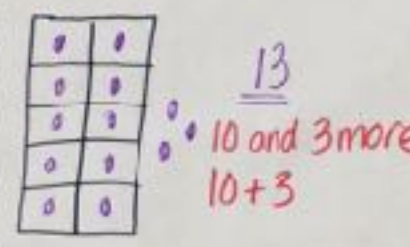


Five Frames!!

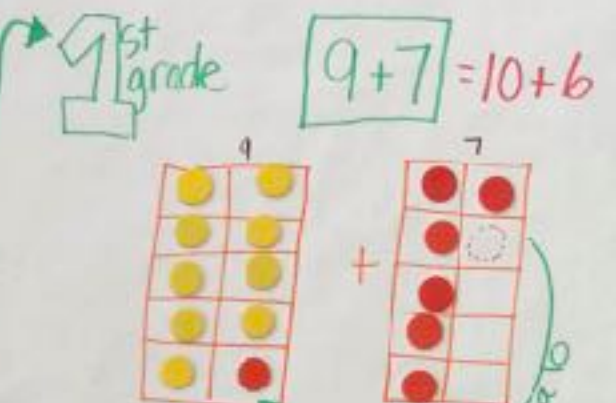
Subtracting 4-3



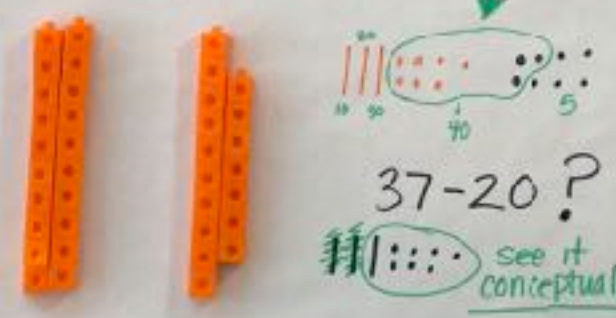
CONTEXT!!



UNITIZING!!
1 ten = 10 ones

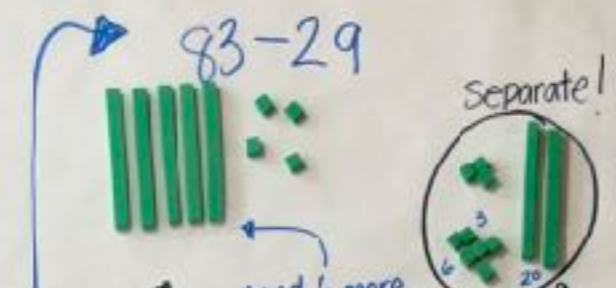


1st grade 37+8 = 40+5

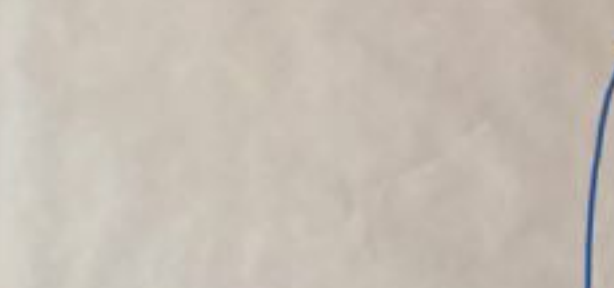


2nd grade 48+33 = 81
make a 10
48 = 40 + 8
+ 33 = 30 + 3
70 + 11 = 81
70 + 10 + 1 = 81
#shortcut

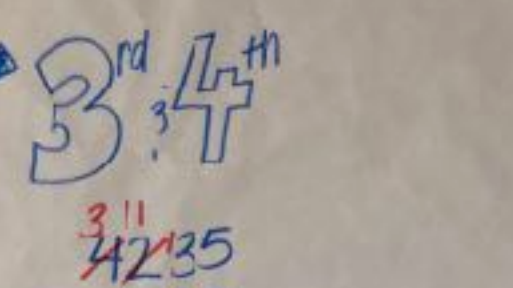
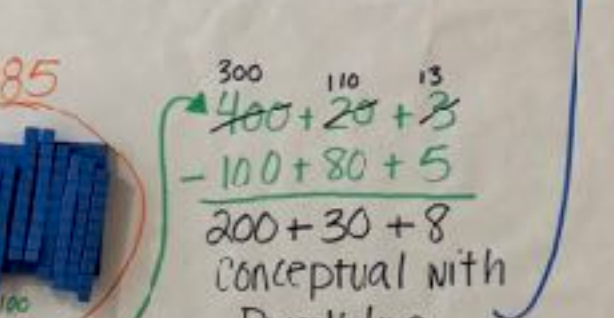
Partial Sums → Partial Products



83-29 = 54
need 6 more
83 = 80 + 3
- 29 = 20 + 9
50 + 4
Place Value



300 100 13
400 + 20 + 3
- 100 + 80 + 5
200 + 30 + 8
Conceptual with Place Value



3rd 4th
4235
- 1672
2563
3000 1000
4000 + 200 + 30 + 5
- 1000 + 600 + 70 + 2
2000 + 500 + 60 + 3

Built through models
#Visualize

Module Breakdown

Part 1: Launch the Concept

Part 2: Connect and Extend Thinking

Week 3: Building Fluency



Games for Fluency Within 20





Sum It Up



Cover the Strip



Sum or Difference



Open Middle Problem



Open Middle

Directions: Using the digits 0-9, fill in the boxes below to create the smallest difference between the two equations. Each box must be used and a digit can only be used one time.

—

+



Open Middle

Directions: Using the digits 0-9, fill in the boxes below to create the smallest difference between the two equations. Each box must be used and a digit can only be used one time.

$$\boxed{1} \boxed{8} \boxed{5} - \boxed{6} \boxed{7}$$

$$\boxed{3} \boxed{4} \boxed{2} + \boxed{9} \boxed{0}$$



Sounds of Imaging Number





No Peeking



Target 999



MAKING SENSE SERIES

Progression of a Number Line in K-2

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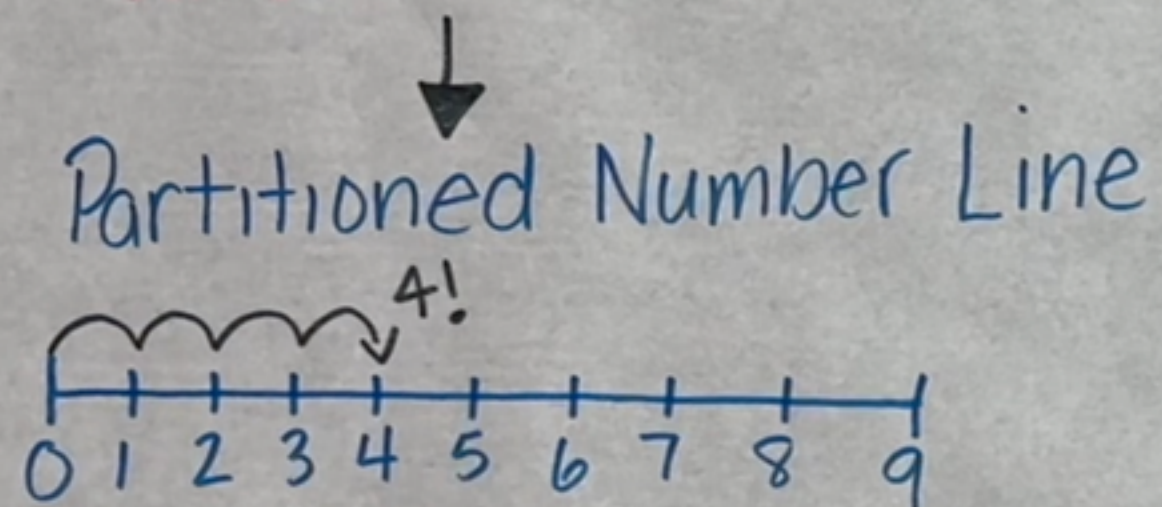


The Progression of a Number Line

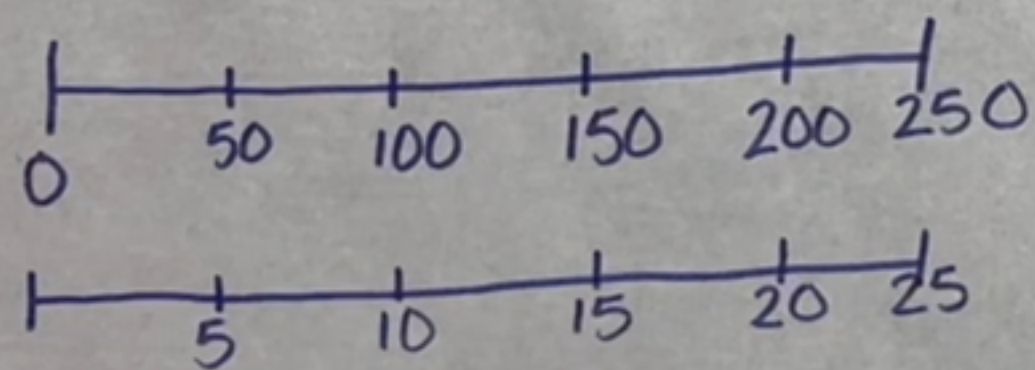
0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

Number Path

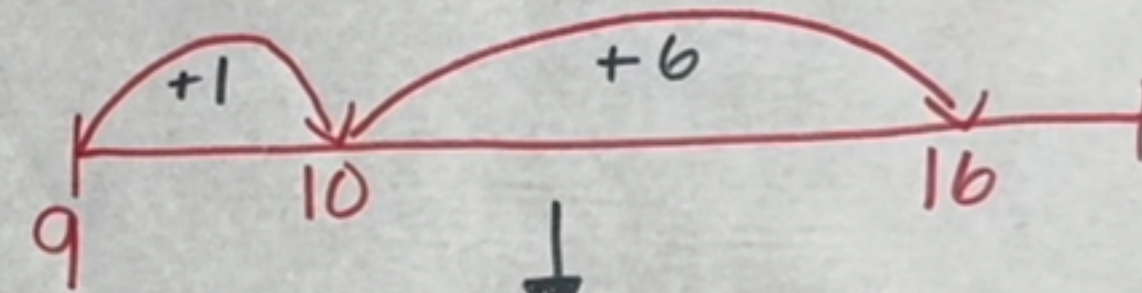


Partitioned with Benchmarks

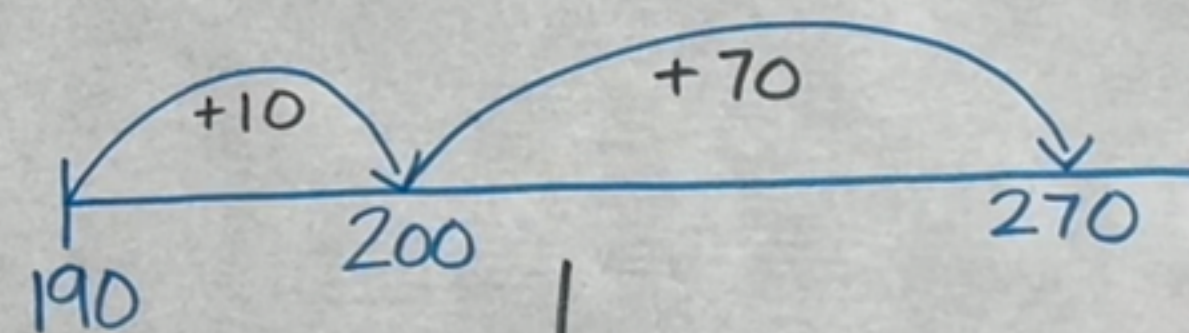


Operations with Number Lines

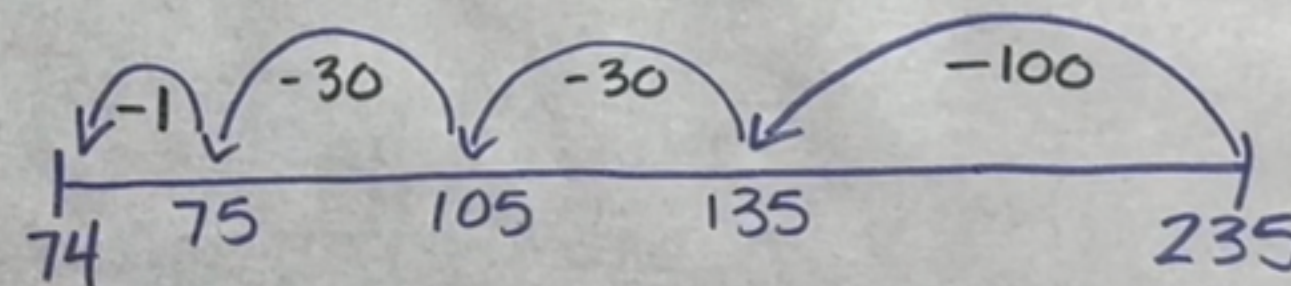
$$9 + 7 = 9 + (1 + 6)$$



$$190 + 80$$



$$235 - 161$$





What's the Middle

What's the Middle?

344

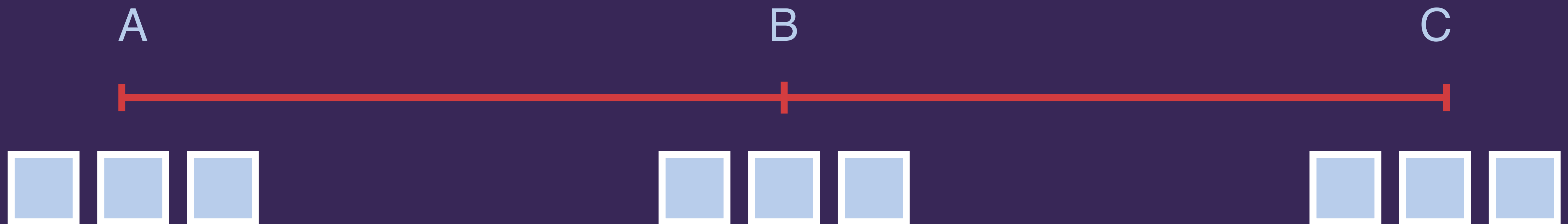
?

788



Open Middle

Directions: Using the digits 0-9, fill in the boxes below so that Point *B* on the number line, is exactly in the middle of Points *A* and *C*. Each box must be used and a digit can only be used one time.





Counting Collections

Find a Friend

Coming Up...

Week 1: Whole Number and Place Value

Week 2: Building Fluency through 1000 (Add & Sub)

Week 3: Building Fluency through 100 (Mult & Div)

Week 4: Building Fluency through 1000 (Mult & Div)

Week 5: Extending Place Value and Operations (Decimals)

Week 6: Number Statistical Reasoning



Foundations of Numerical Reasoning in Grades 3-5

Module 2: Building Fluency Through 1000 (Add & Sub)

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