# Foundations of Numerical Reasoning in Grades K-2

## Module 1: Early Number and Counting

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# 6 Week Module Breakdown

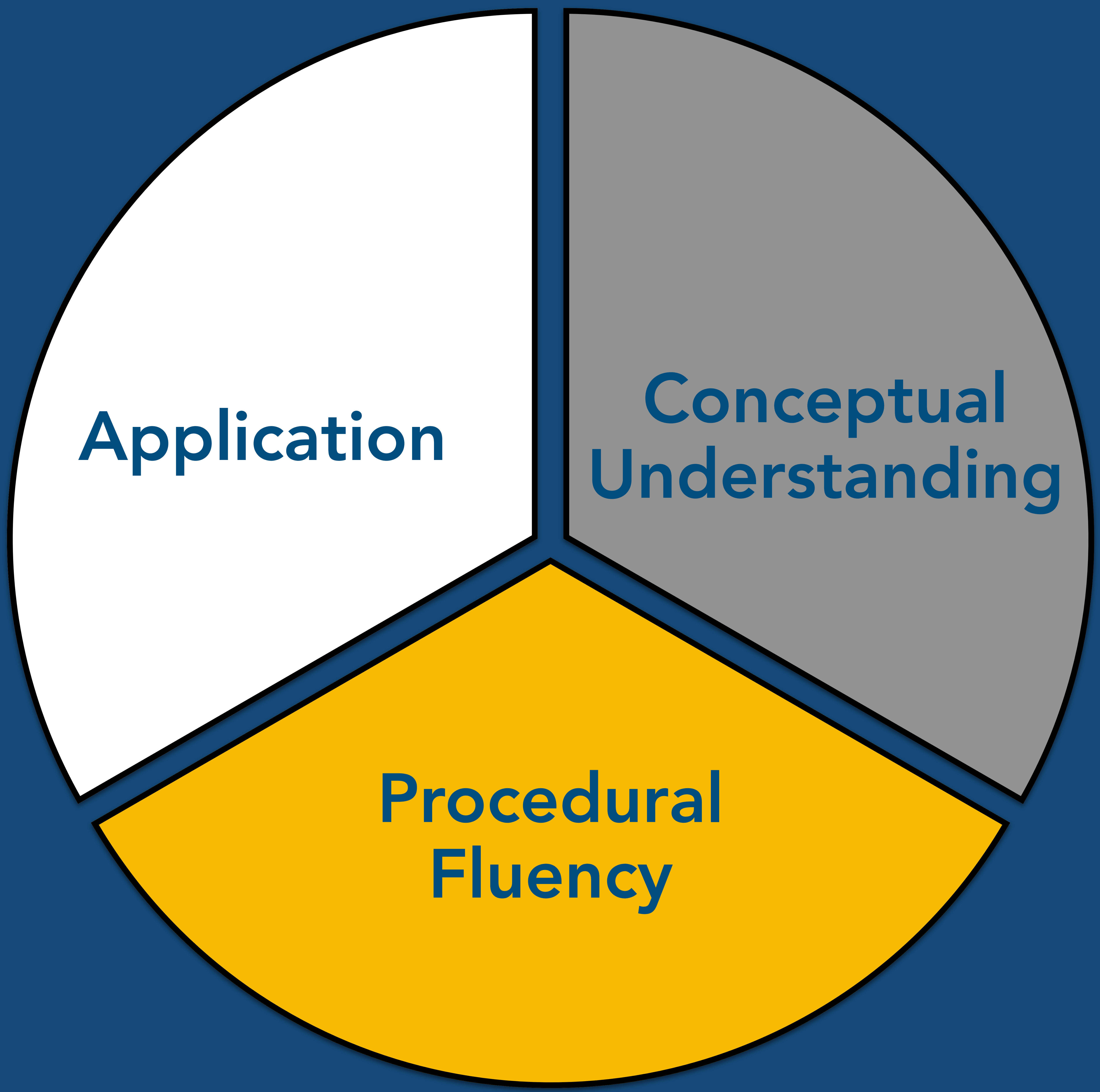
**Week 1:** Early Number and Counting
**Week 2:** Building Fluency through 20
**Week 3:** Building Fluency through 120
**Week 4:** Building Fluency through 1000
**Week 5:** Number and Measurement
**Week 6:** Number and Statistical Reasoning

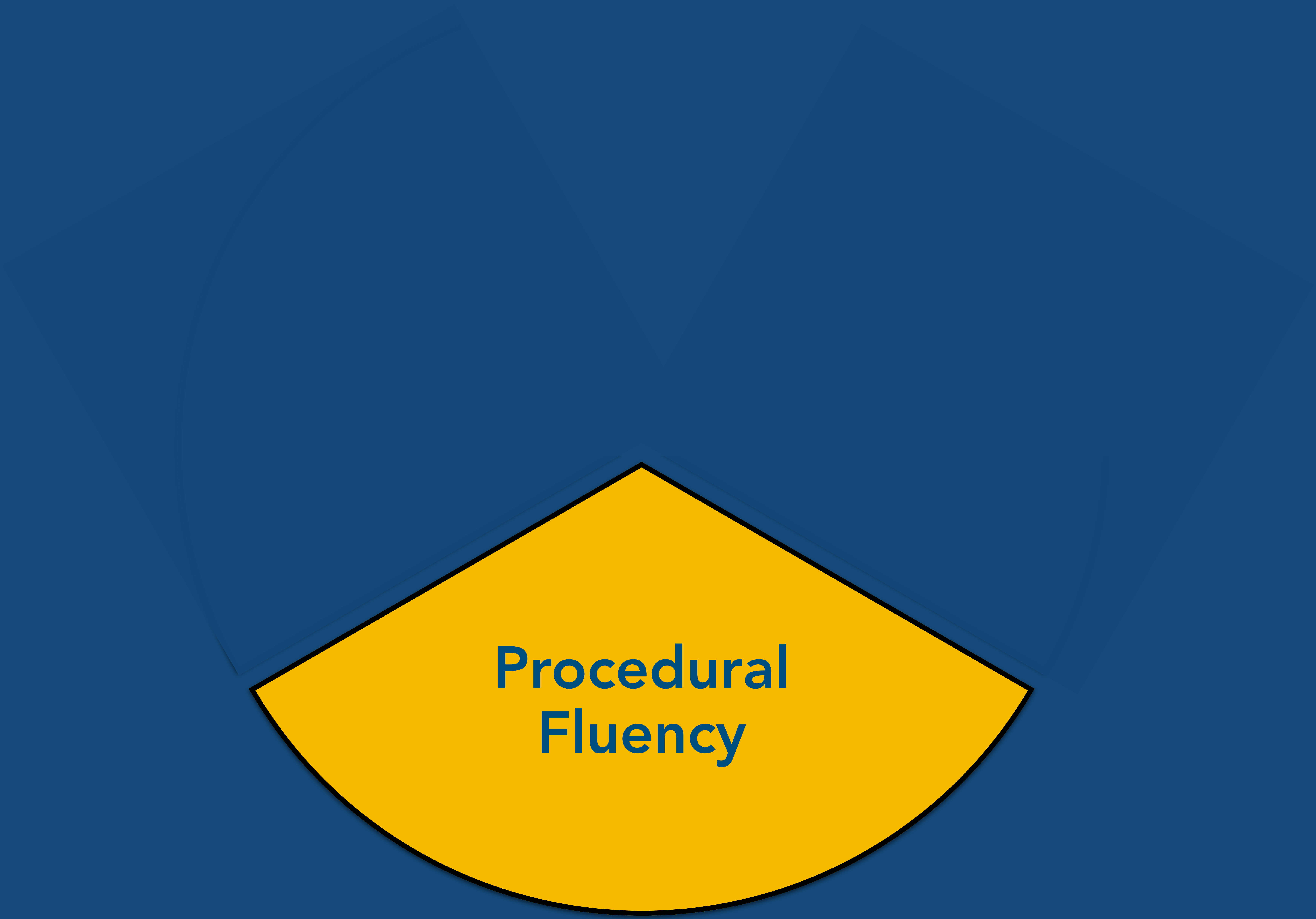
# Module Breakdown

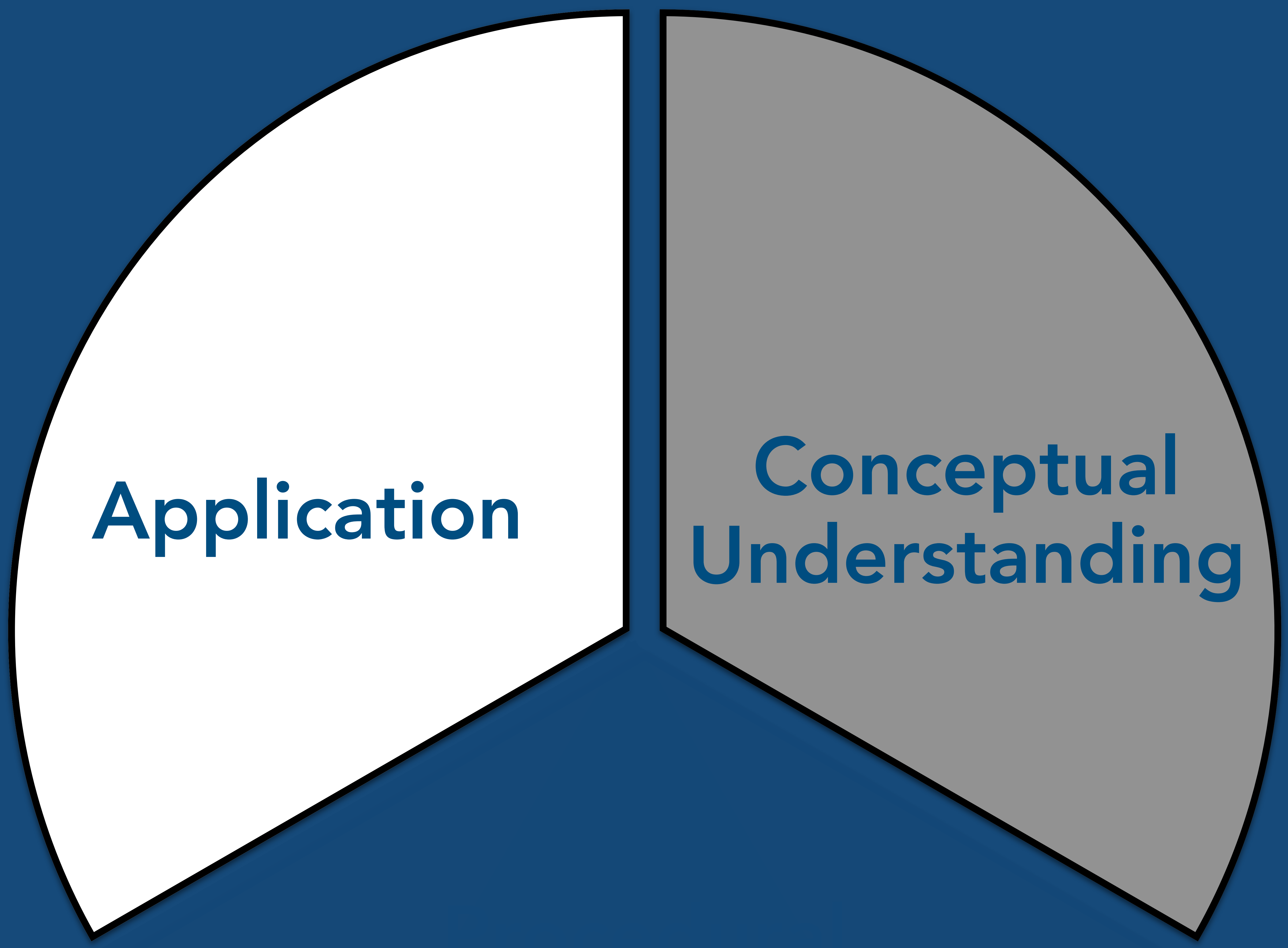
**Part 1:** Launch the Concept

**Part 2:** Connect and Extend Thinking

**Part 3:** Building Fluency


Application
Conceptual Understanding
Procedural Fluency


Procedural Fluency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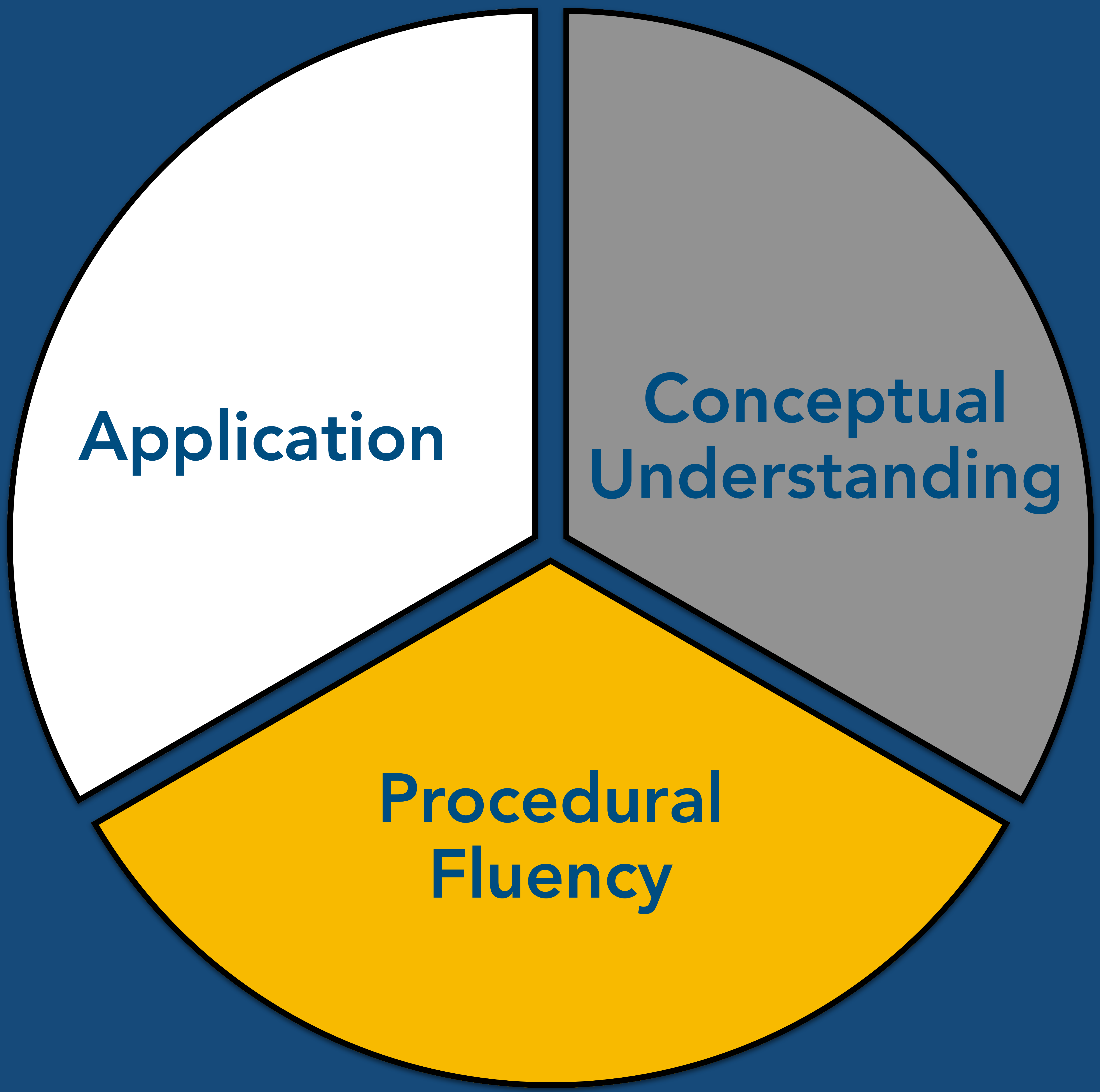
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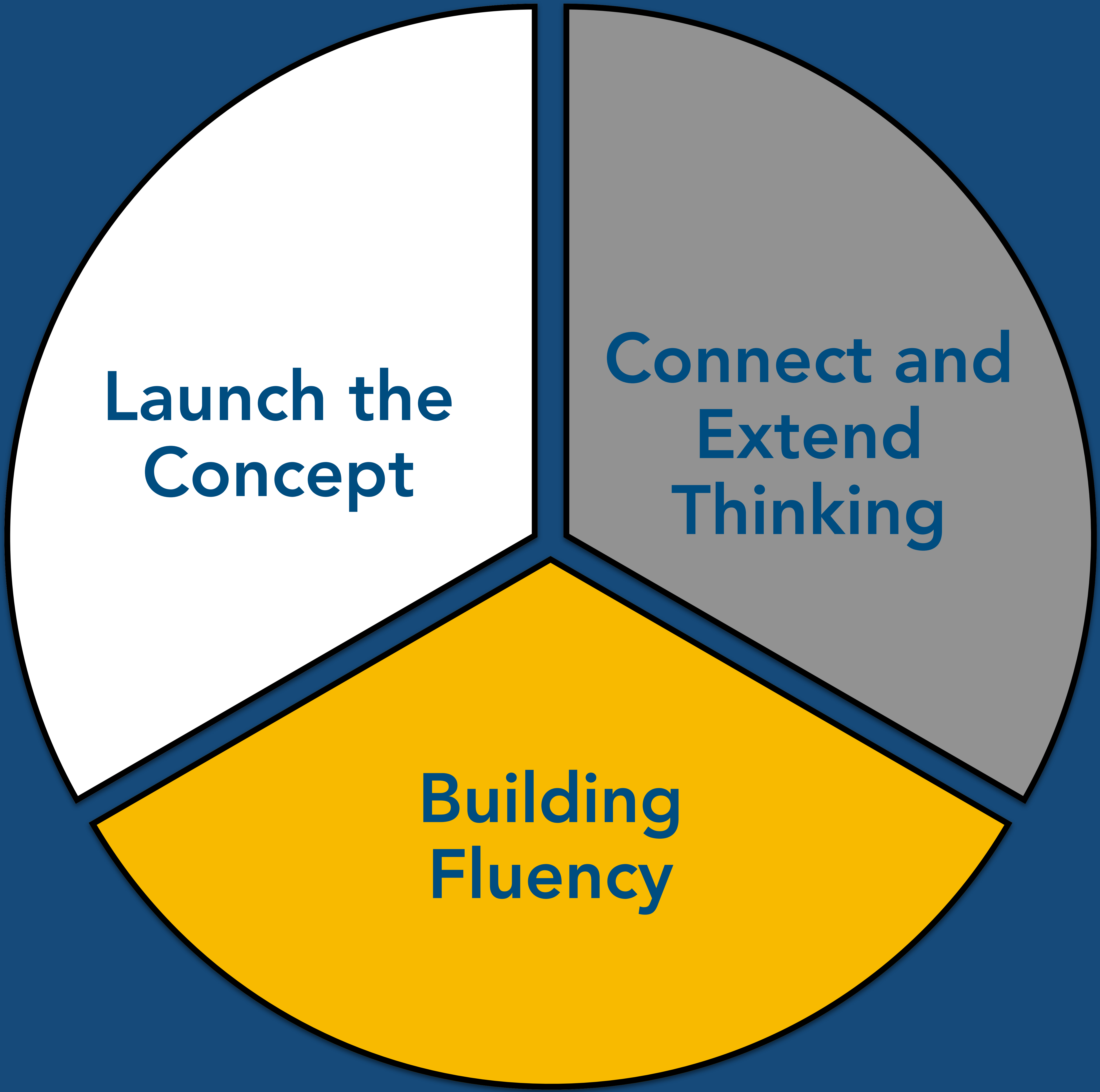
Launch the Concept
Connect and Extend Thinking
Building Fluency

# Housekeeping

Handouts and Recording sheets

DO NOT LOOK

PRINT

Hitting Pause to work through some tasks or activities

# 6 Week Module Breakdown

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# Module 1

**Part 1:** Launch the Concept

**Part 2:** Connect and Extend Thinking

**Part 3:** Building Fluency

# 3-Act Task

Name: ______________________________________________

1. What did you notice?

2. What do you wonder?

3. Main Question:

4. Estimate:

5. What information do you need?

6. Show your thinking:

NAME:________________________________________

#1 Estimate

#2-Show your thinking

#3-Answer

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# Act 1


# Notice?


# Notice?
# Wonder?

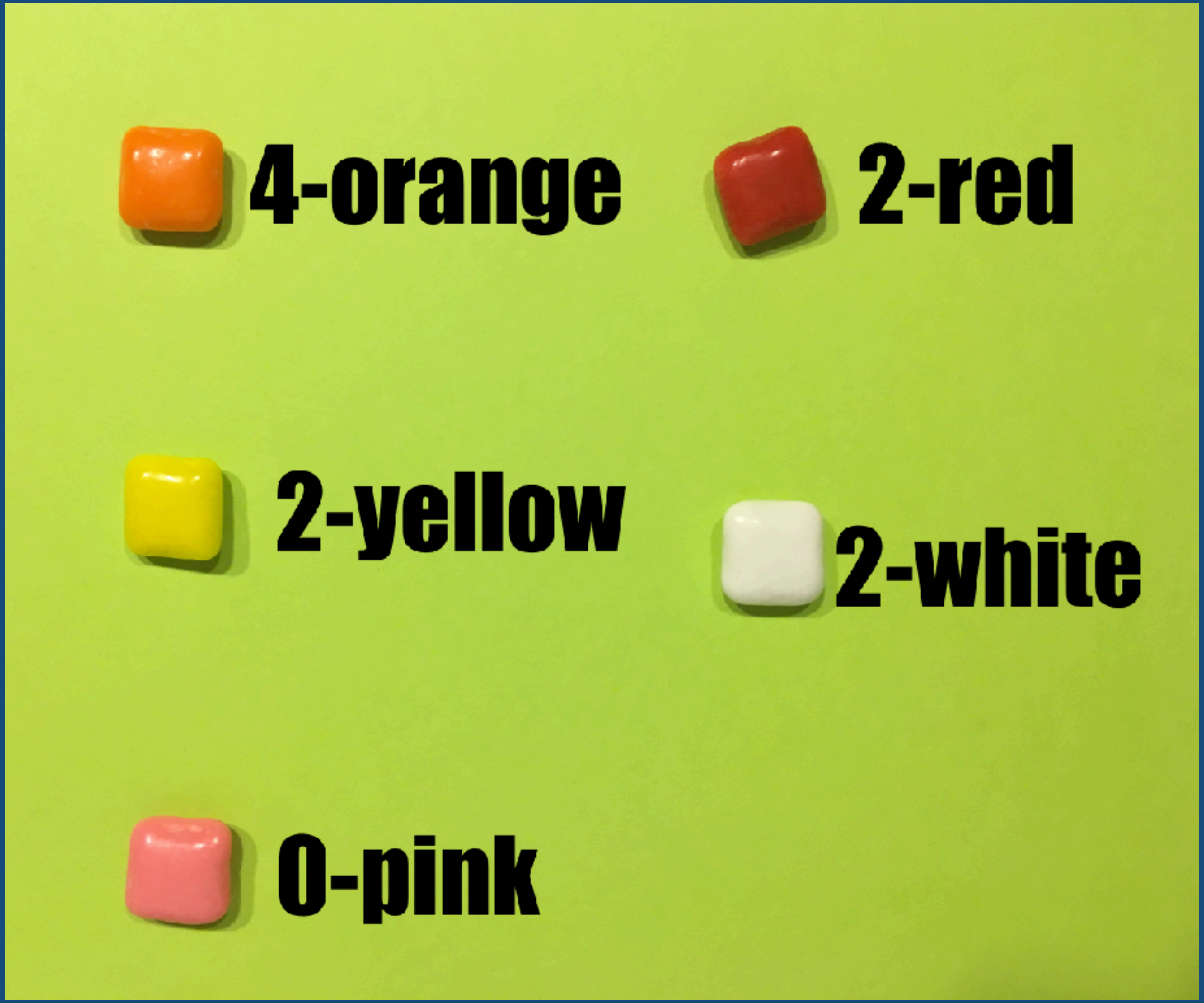

# How many candies are in my hand?


# Estimate

How many candies are in my hand?

# Act 2


# What information would you link to know?




# How many of each color

# Act 3


# Reveal

# 3-Act Tasks


**Act 1:**

- Real world problem or scenario presented
- What do you notice? What do you wonder?
- Make estimates

**Act 2:**

- Identify missing variables and missing variables to solve
- Define solution path using variables

**Act 3:**

- Solve and interpret results of the solution
- Validate answer

## Most asked questions:

- How often should we use 3-Act Tasks?
- When should we use 3-Act tasks? How do they fit into the scope of a unit?
- How long does one task usually take?
- What if we don't have the time?

# Orchestrating Discussions

*Five practices constitute a model for effectively using student responses in whole-class discussions that can potentially make teaching with high-level tasks more manageable for teachers.*

Margaret S. Smith, Elizabeth K. Hughes, Randi A. Engle, and Mary Kay Stein

**Margaret S. Smith,** pegs@pitt.edu, is an associate professor of mathematics education at the University of Pittsburgh. Over the past decade, she has been developing research-based materials for use in the professional development of mathematics teachers and studying what teachers learn from the professional development in which they engage. **Elizabeth K. Hughes,** elizabeth.hughes@uni.edu, recently finished her doctorate in mathematics education at the University of Pittsburgh. Her areas of interest include preservice secondary mathematics teacher education and the use of practice-based materials in developing teachers' understanding of what it means to teach and learn mathematics. **Randi A. Engle,** raengle@berkeley.edu, is an assistant professor of mathematics education and the social context of learning at the University of California Berkeley. She is interested in developing practical theories for how mathematics teachers can create discussion-based learning environments that promote strong student engagement, learning, and transfer. **Mary Kay Stein,** mkstein@pitt.edu, is a professor of learning sciences and policy and the director of the Learning Policy Center at the University of Pittsburgh. Her research focuses on instructional practice and the organizational and policy conditions that shape it.

Discussions that focus on cognitively challenging mathematical tasks, namely, those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor, and Resnick forthcoming). Such discussions give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives (NCTM 2000).

Although discussions about high-level tasks provide important







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# Pause II




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# The 5 Practices:

1. **Anticipating** student responses to challenging mathematical tasks;
2. **Monitoring** students’ work on and engagement with the tasks;
3. **Selecting** particular students to present their mathematical work;
4. **Sequencing** the student responses that will be displayed in a specific order and;
5. **Connecting** different students’ responses and connecting the responses to key mathematical ideas.

*MTMS: Vol. 14, No. 9, May 2009-5 Practices for Orchestrating Productive Mathematics Discussions*

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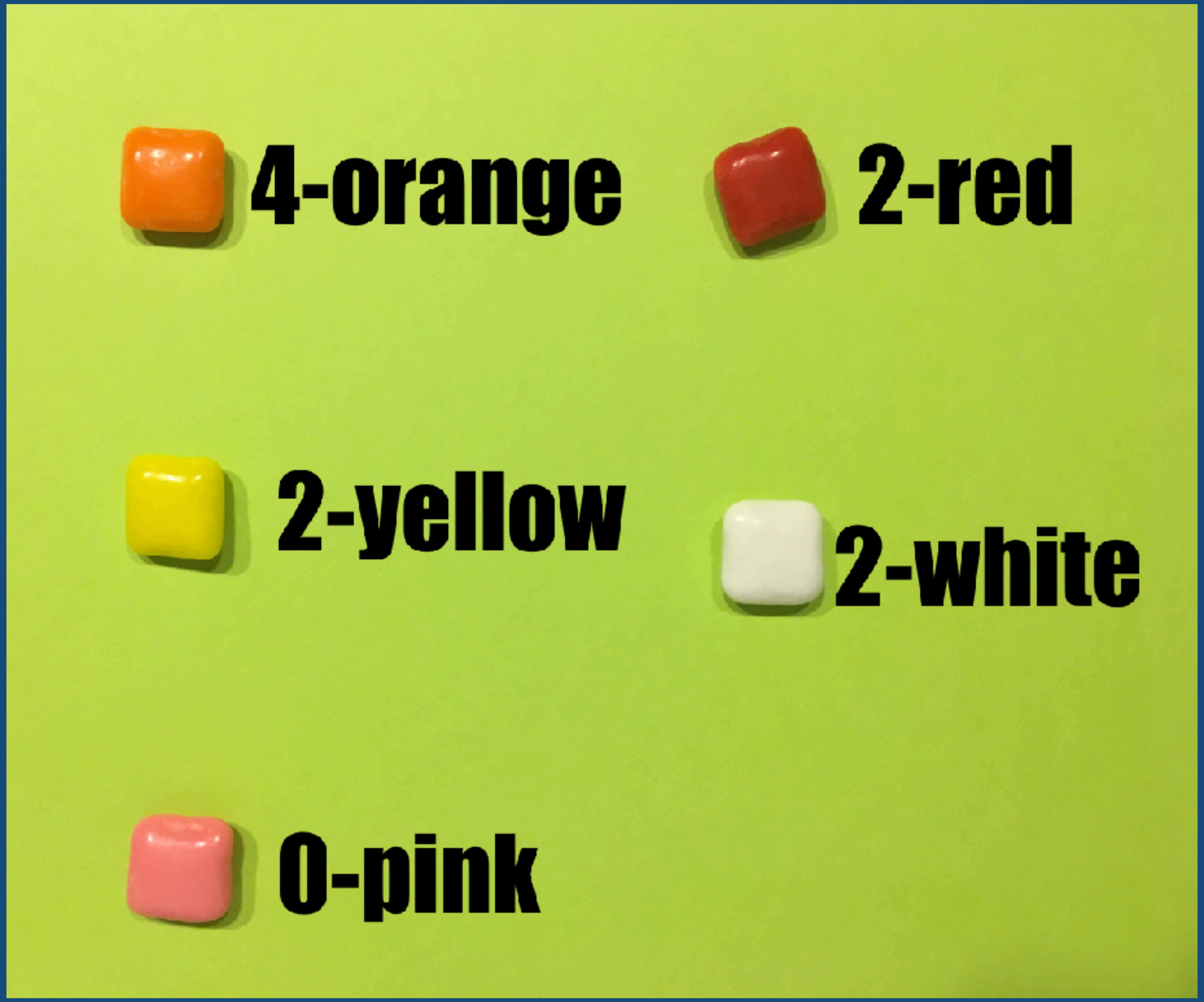
Task Planning Page

| Learning Target: | | |
|---|---|---|
| Questions, Look-Fors and Misconceptions: | | |

| Strategy | Who and What | Order |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

| Scaffolding | Extension | Notes |
|---|---|---|
| | | |

Anticipating → Monitoring → Selecting → Sequencing → Connecting



# Pause

II

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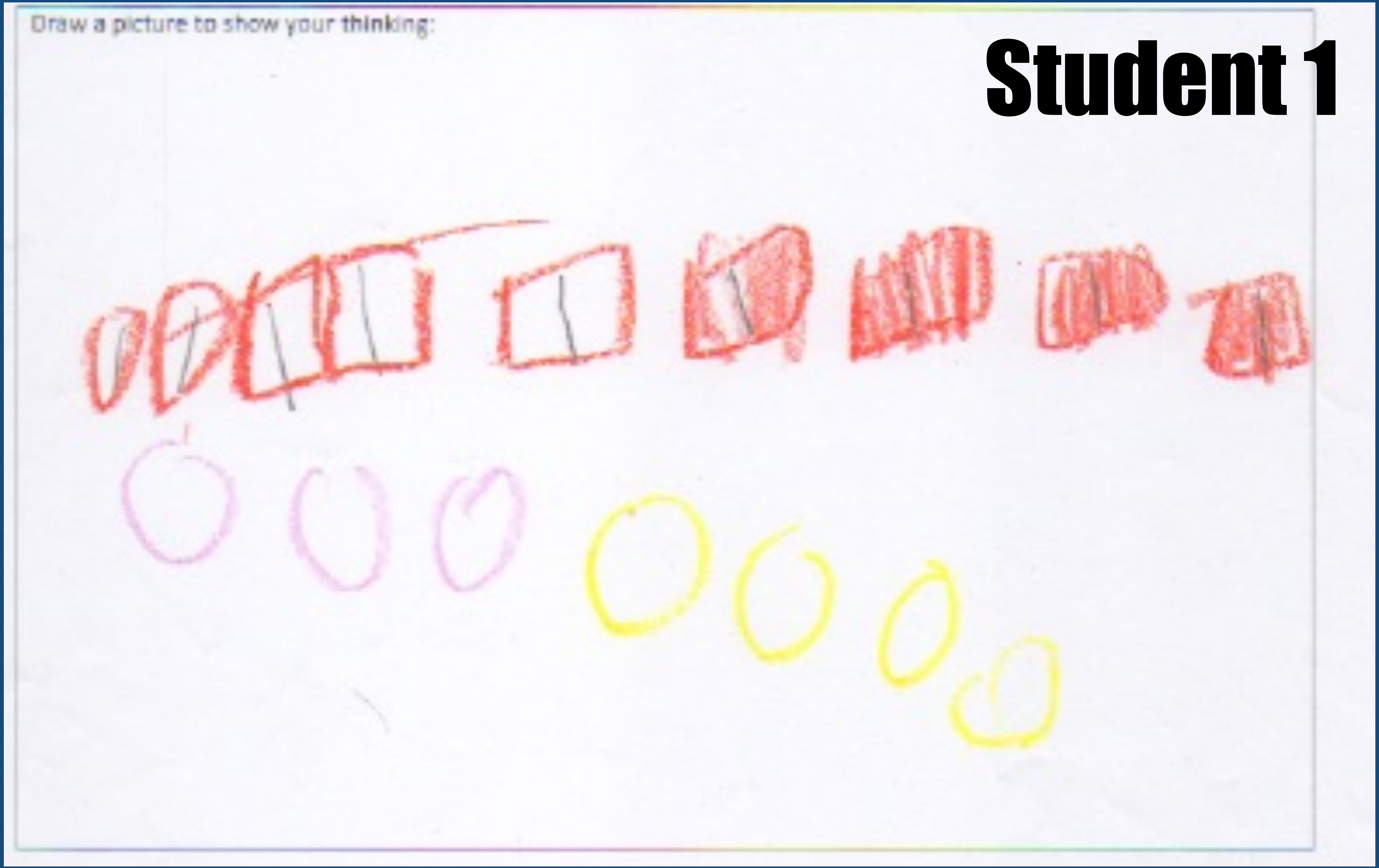
Student 1



Draw a picture to show your thinking:

# Student 2

# Student 3

Draw a picture to show your thinking:

Use numbers to show your thinking:

1 2 3 4 2 6 7 8 9 1 6

Draw a picture to show your thinking:

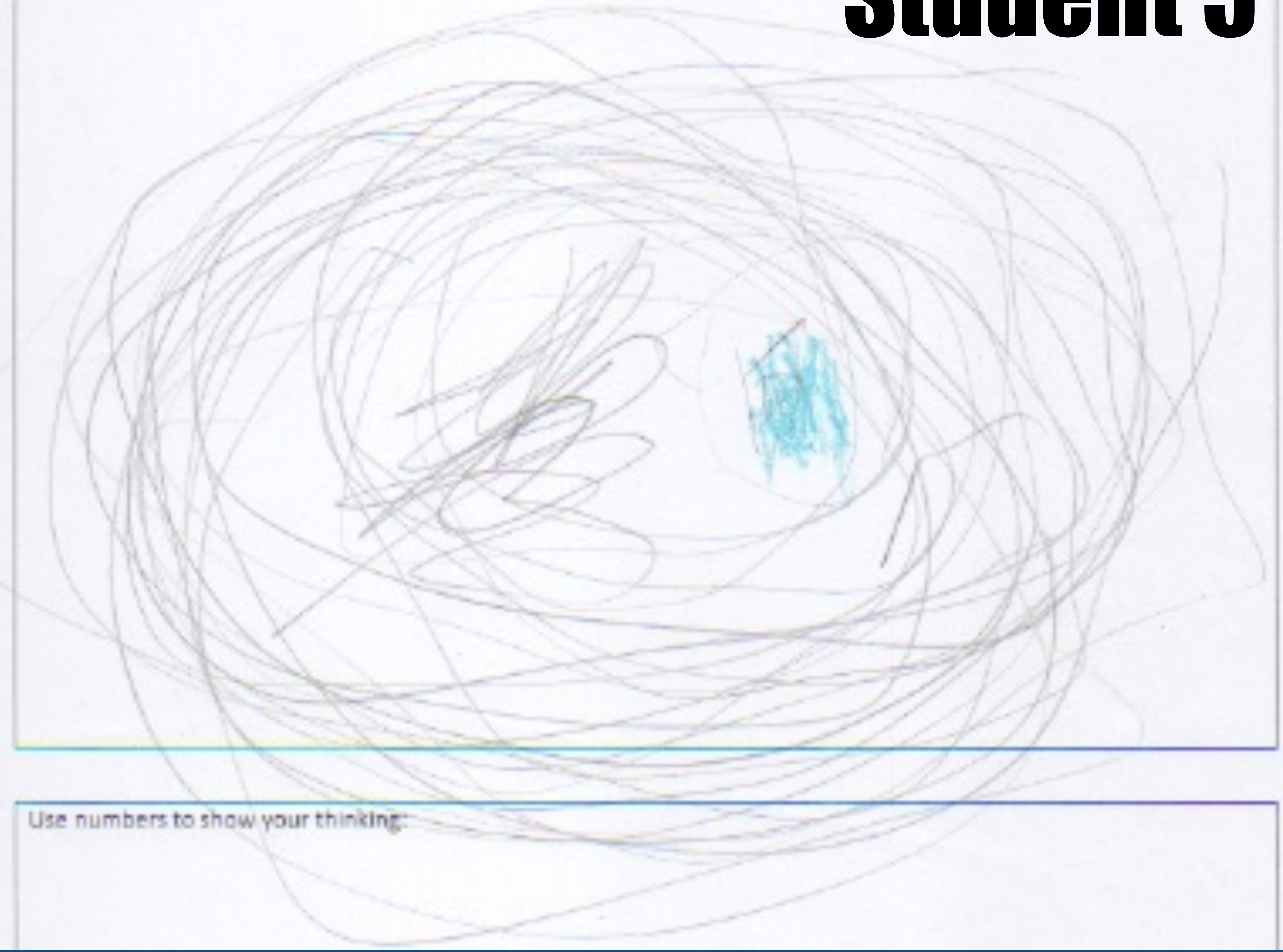
$4+2+2+2+0=$

$10$

**Student 4**

# Student 5

Draw a picture to show your thinking:

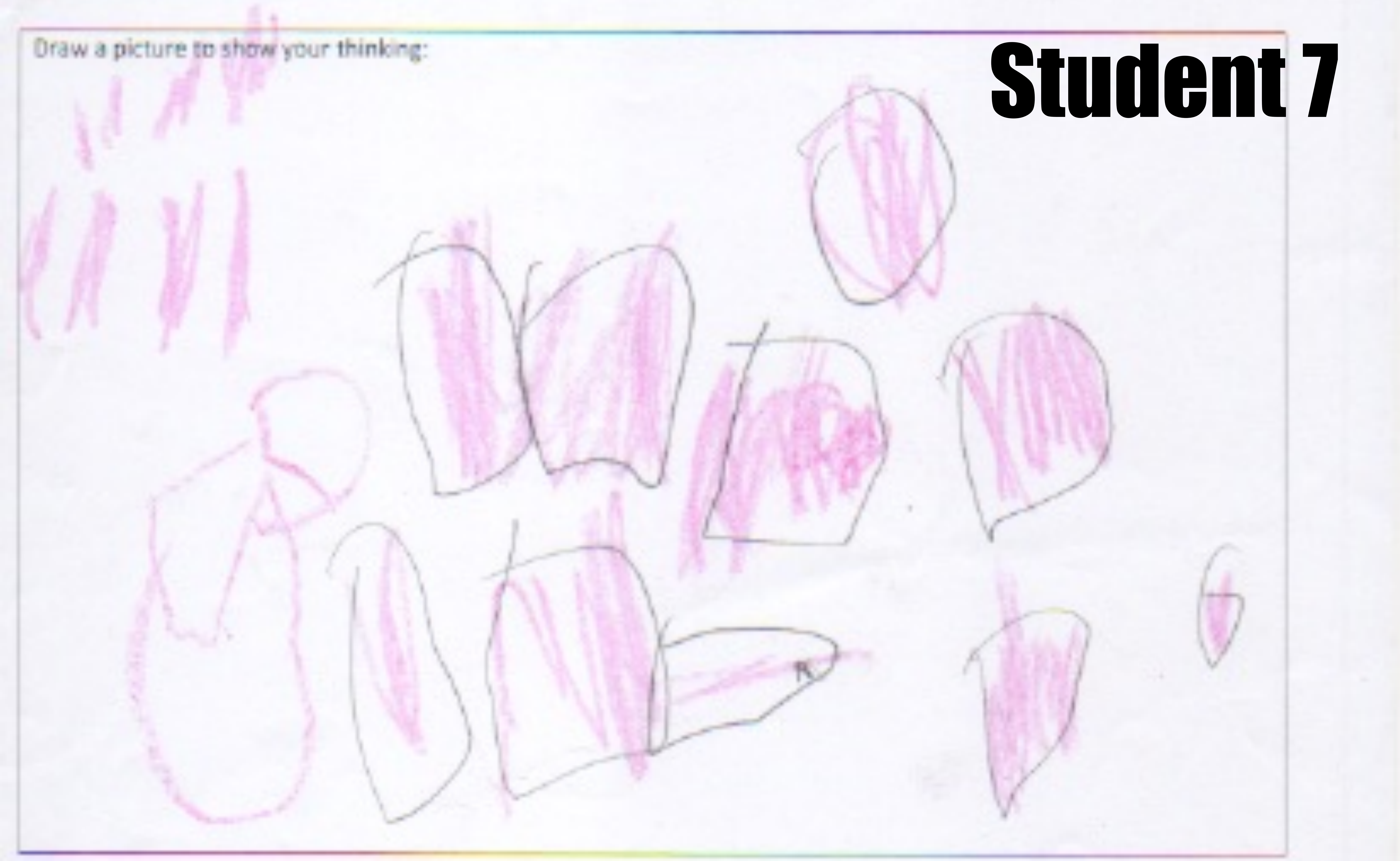


Use numbers to show your thinking:

Draw a picture to show your thinking:

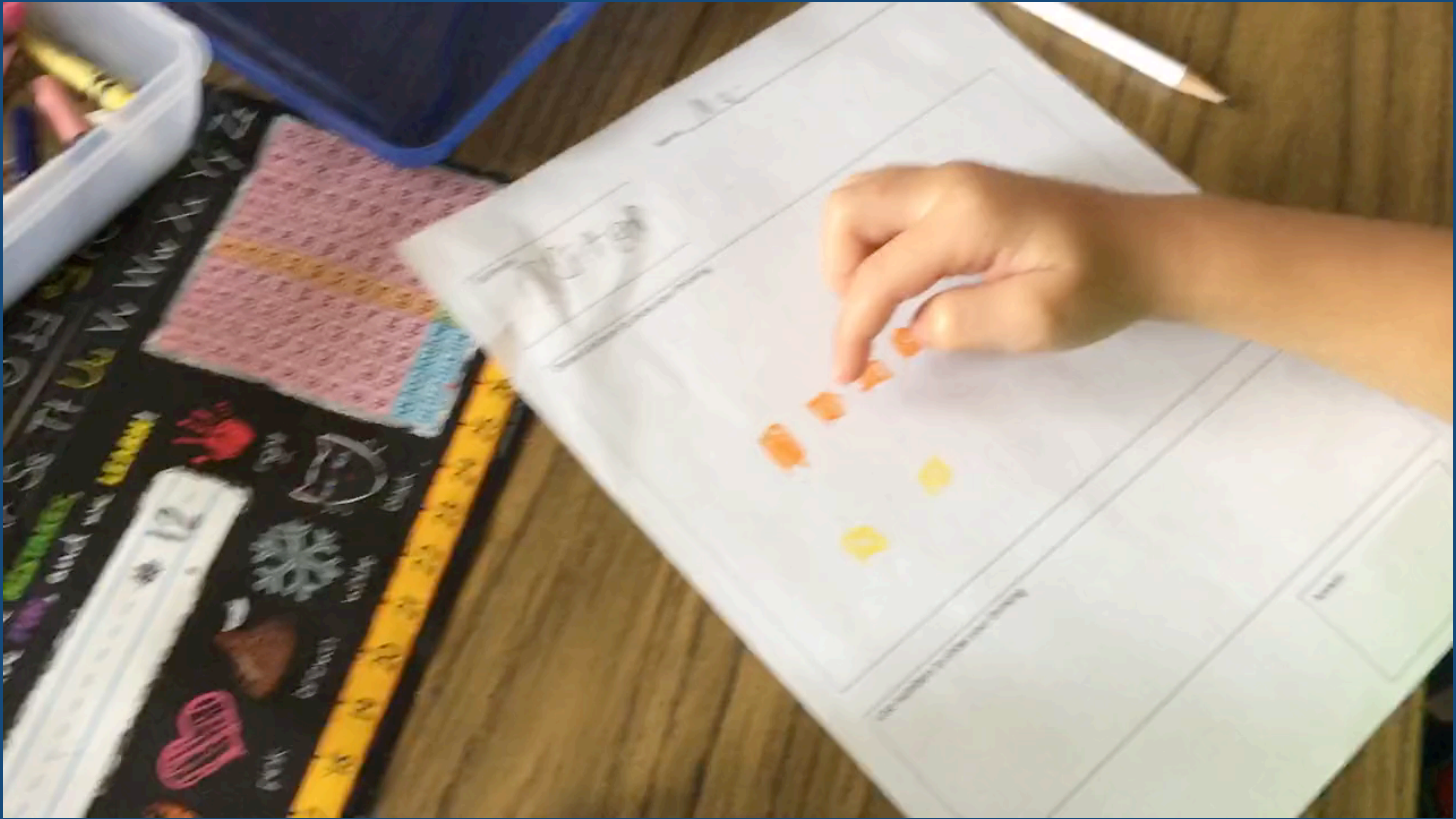
# Student 6

# Student 7



# Student 8

Draw a picture to show your thinking:



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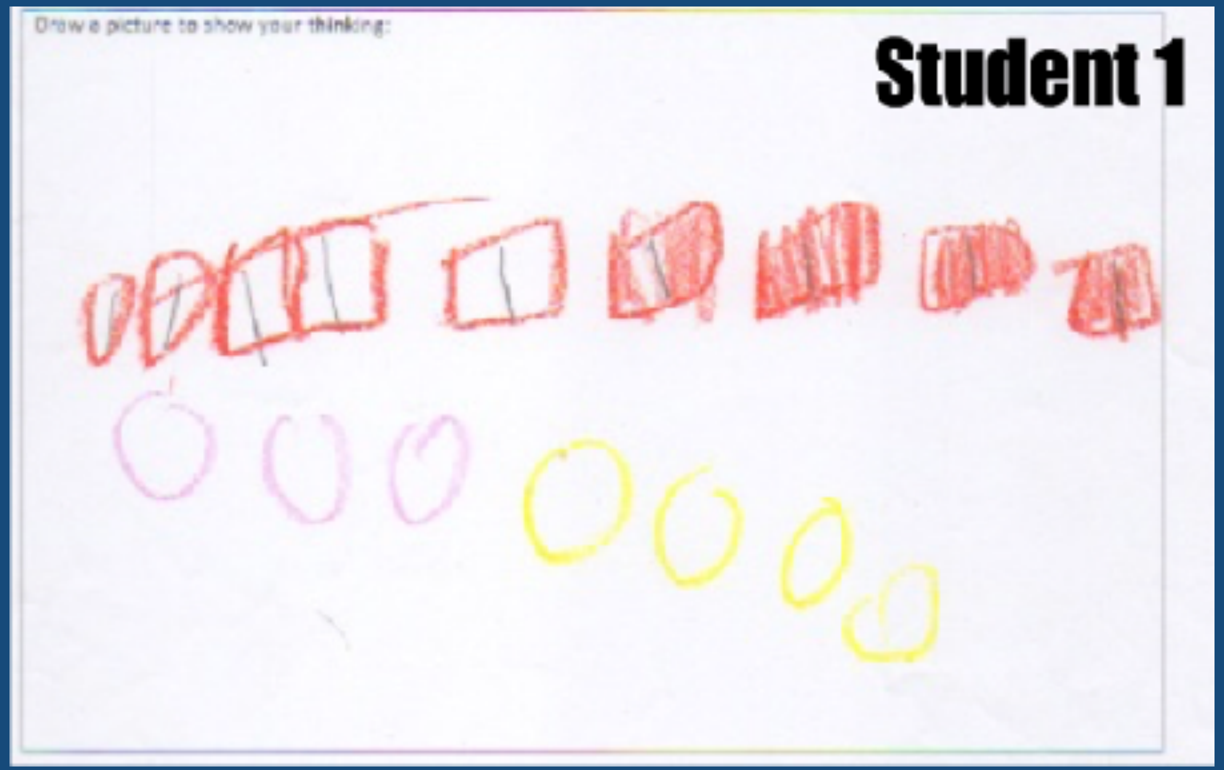
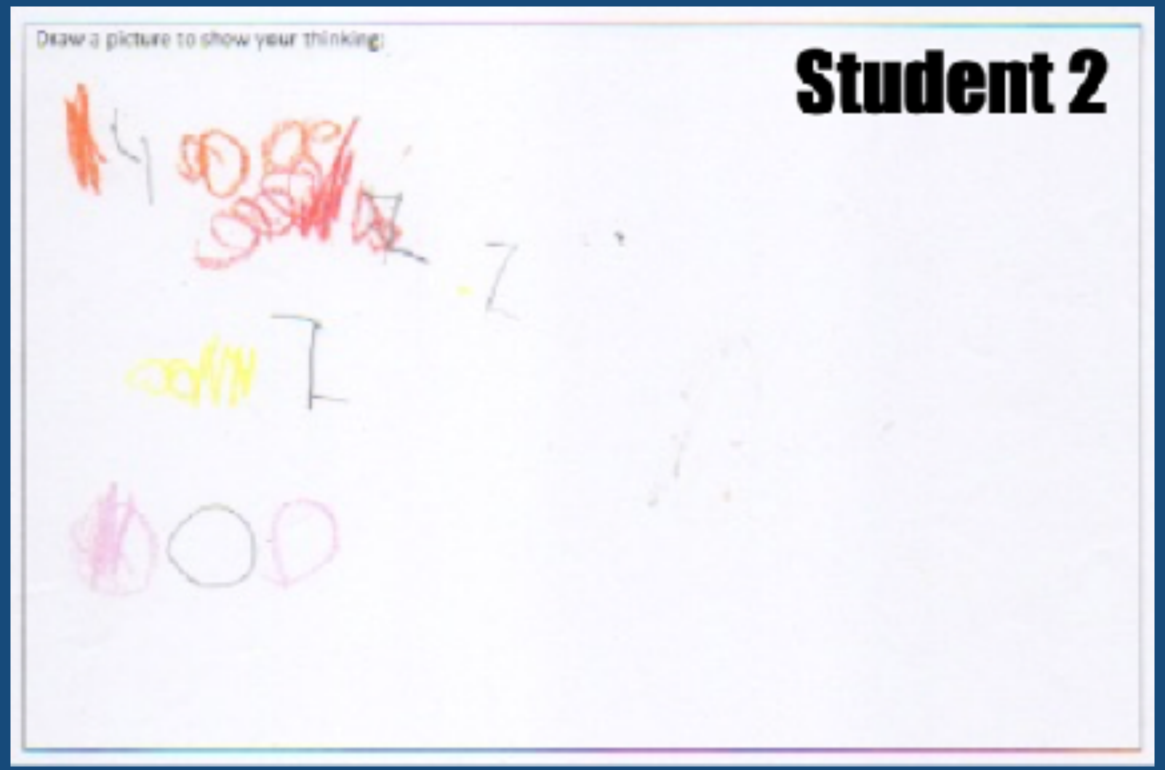
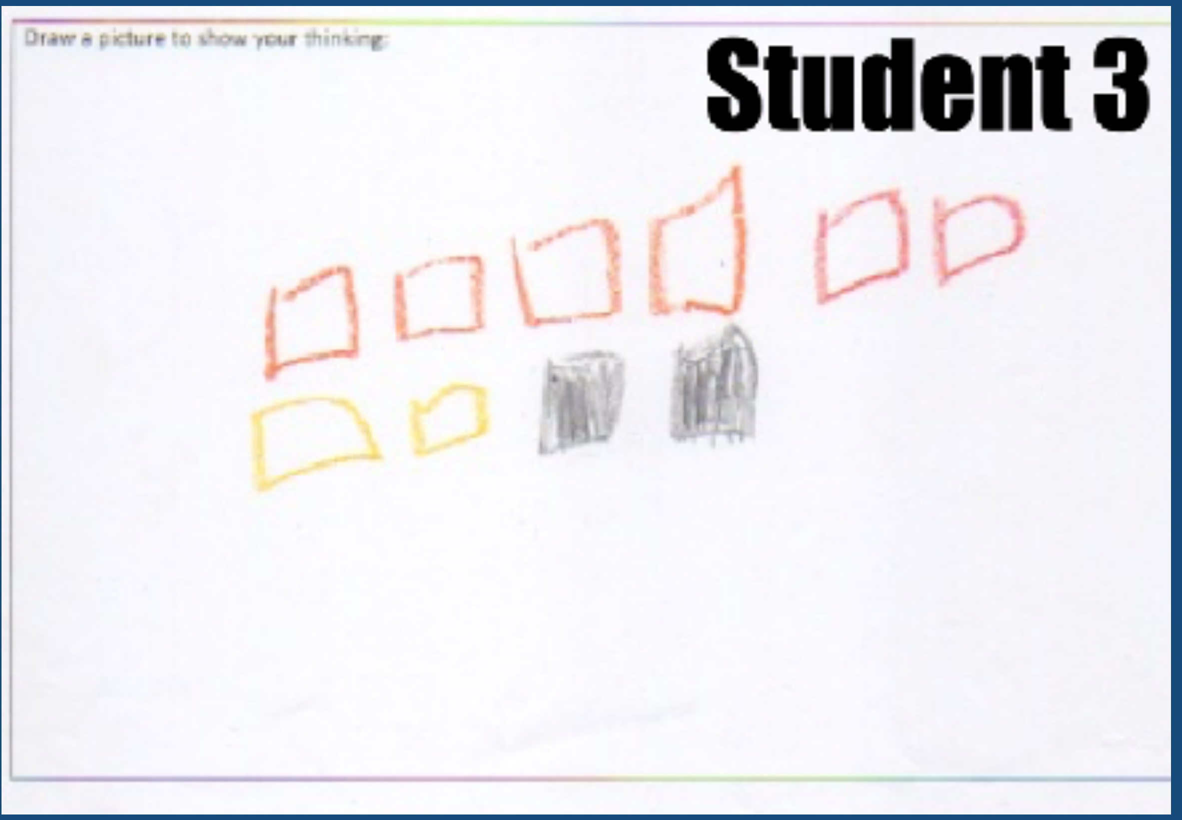
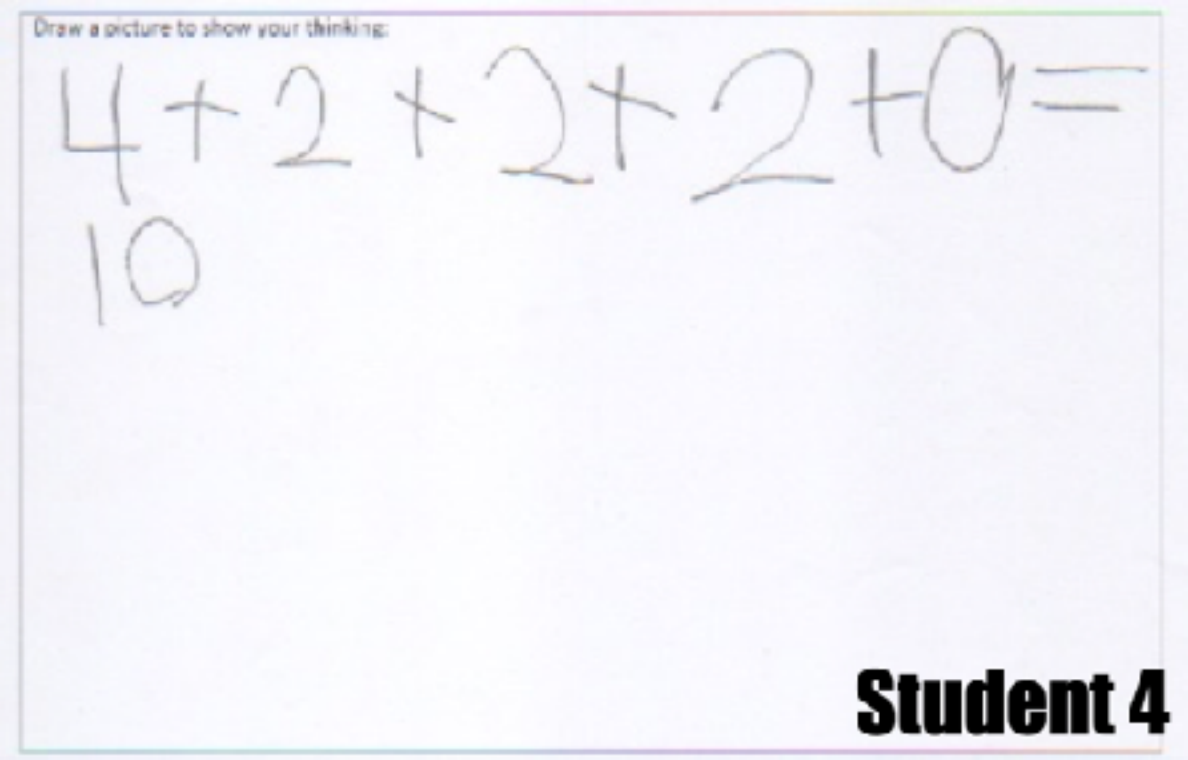
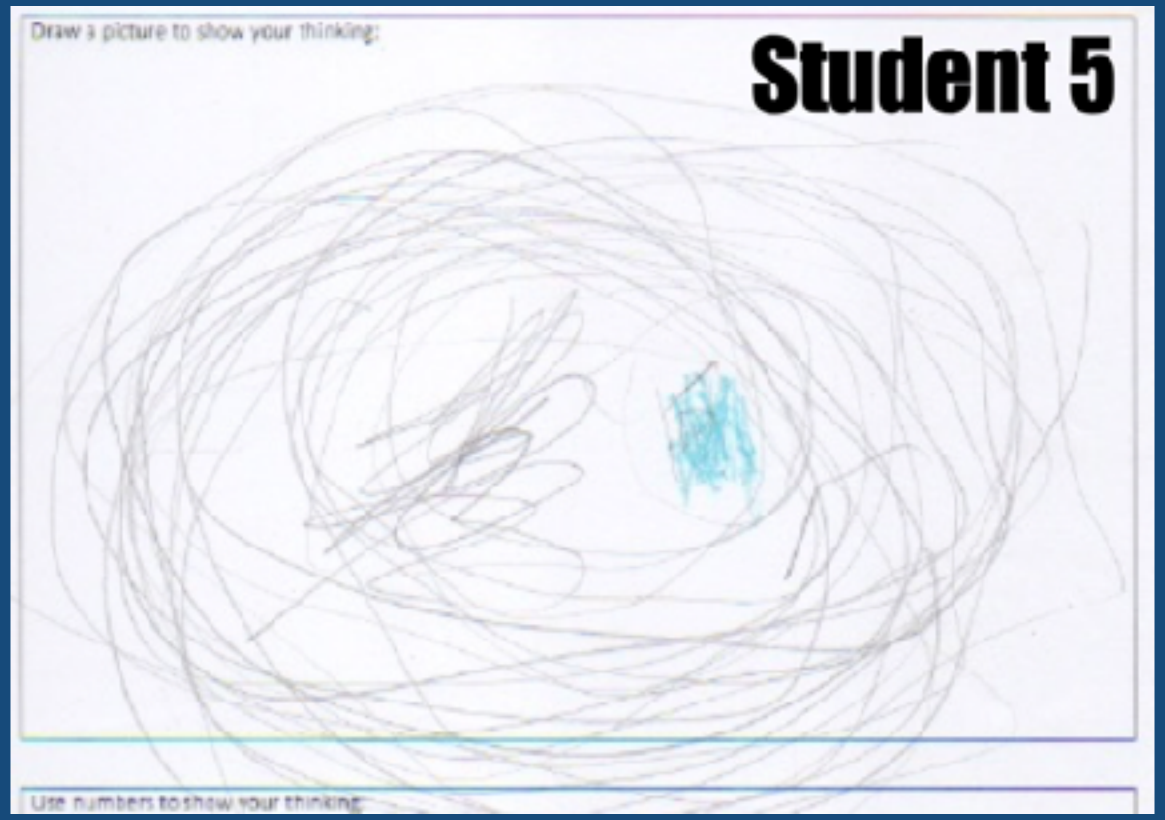
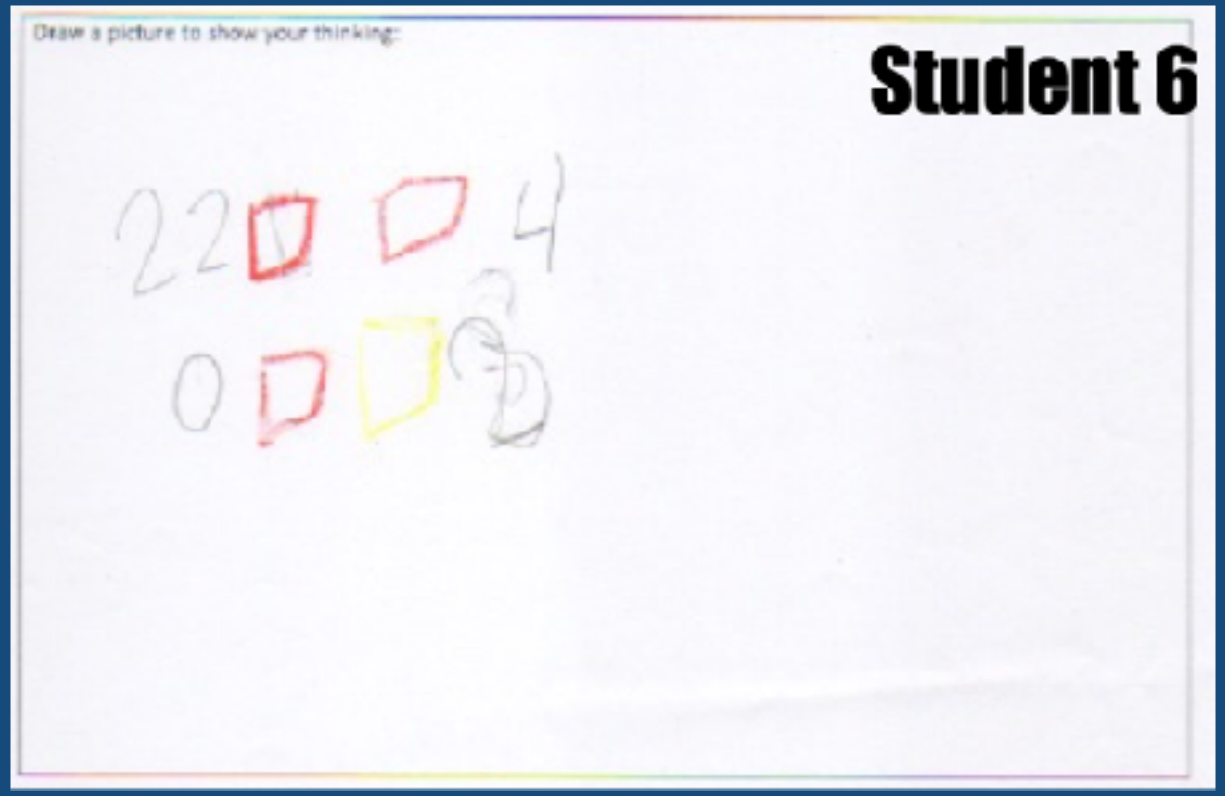


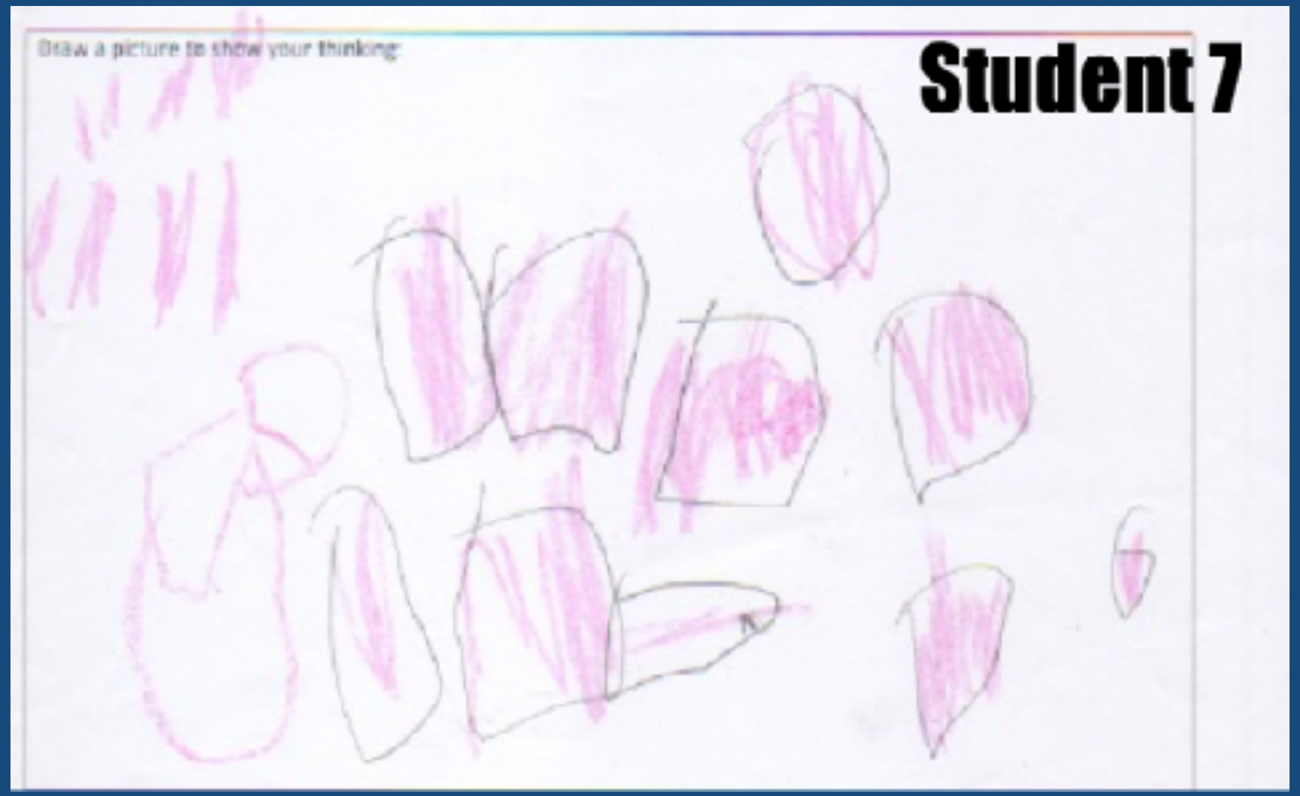



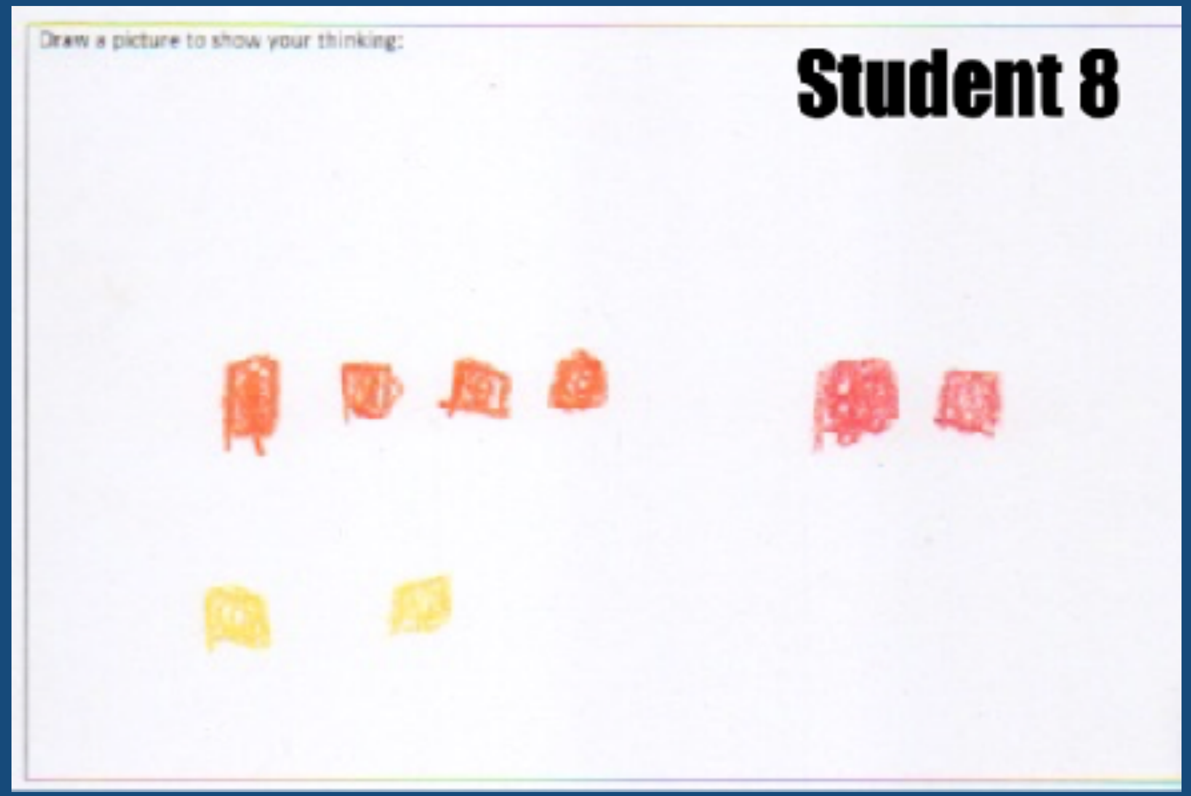



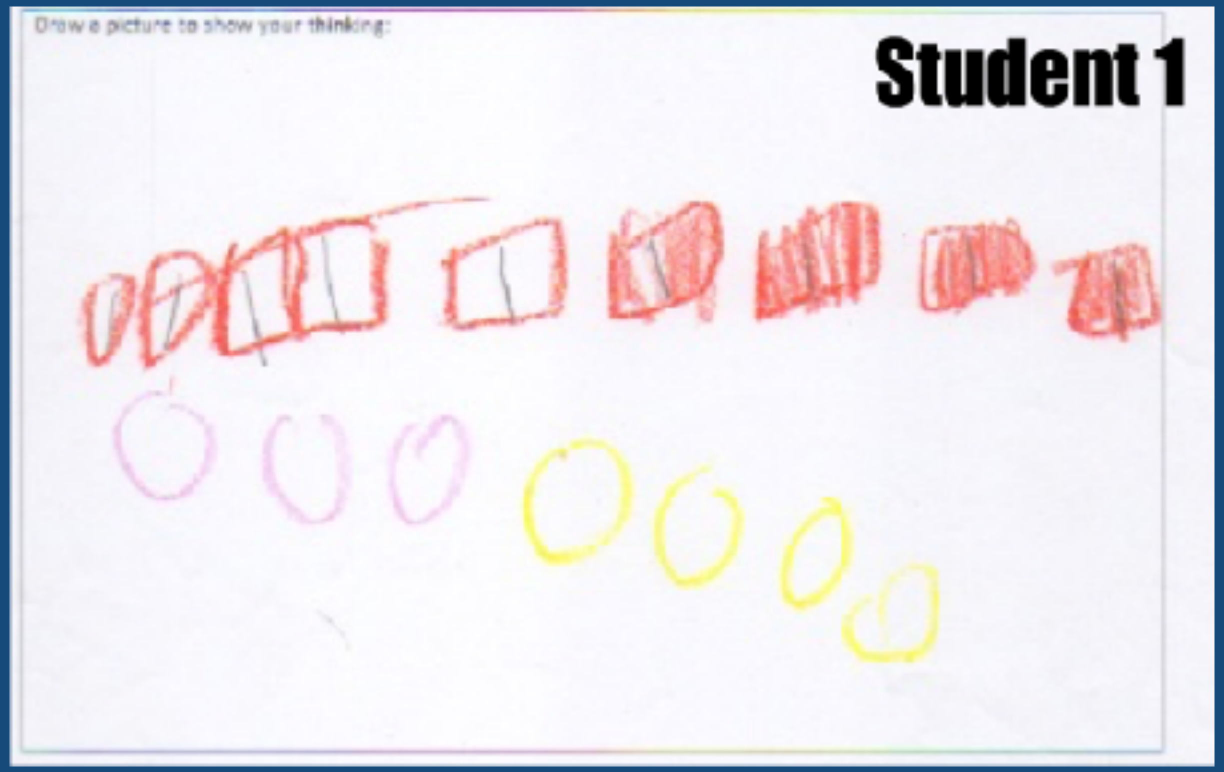



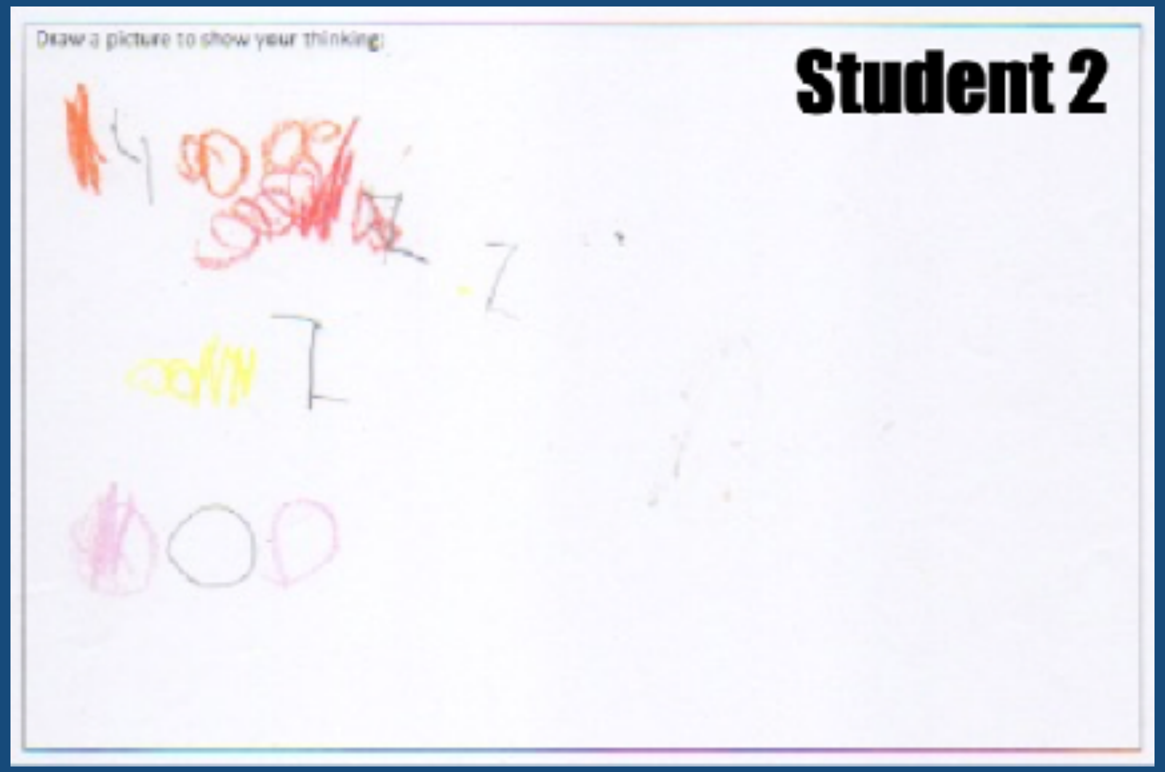



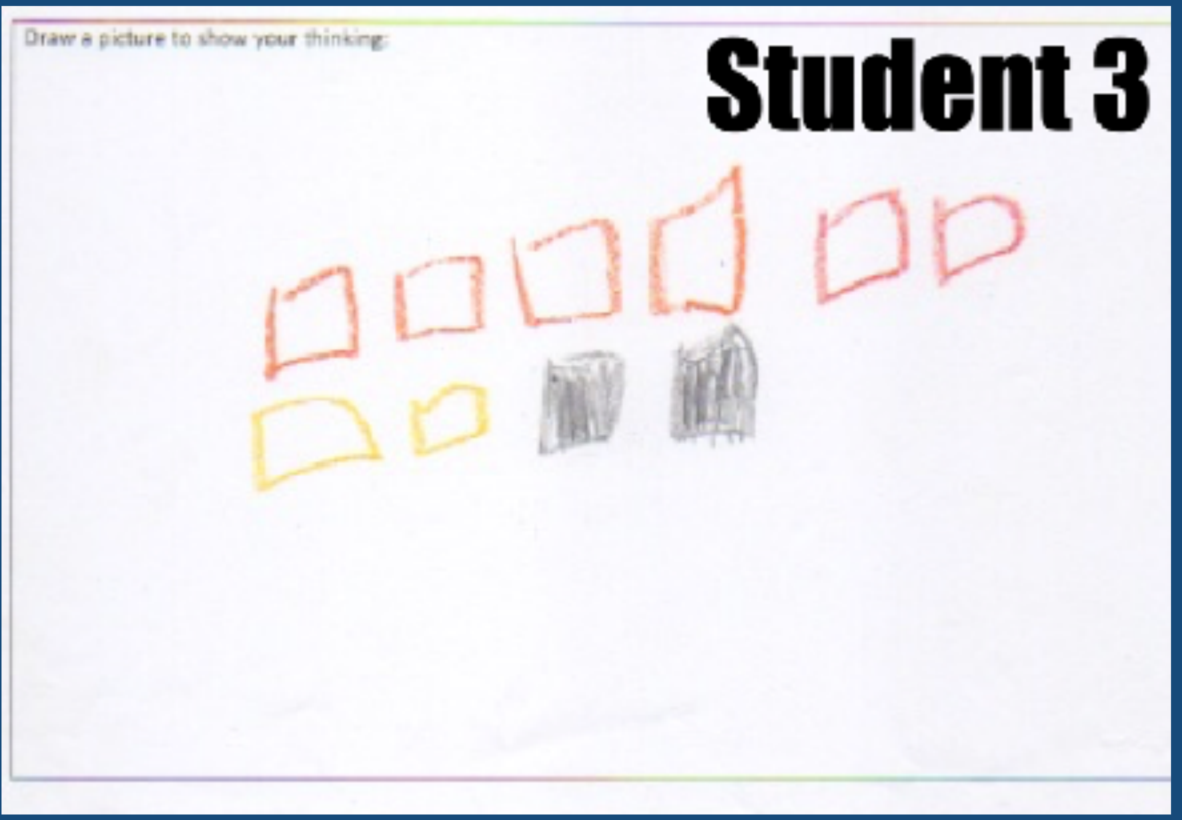



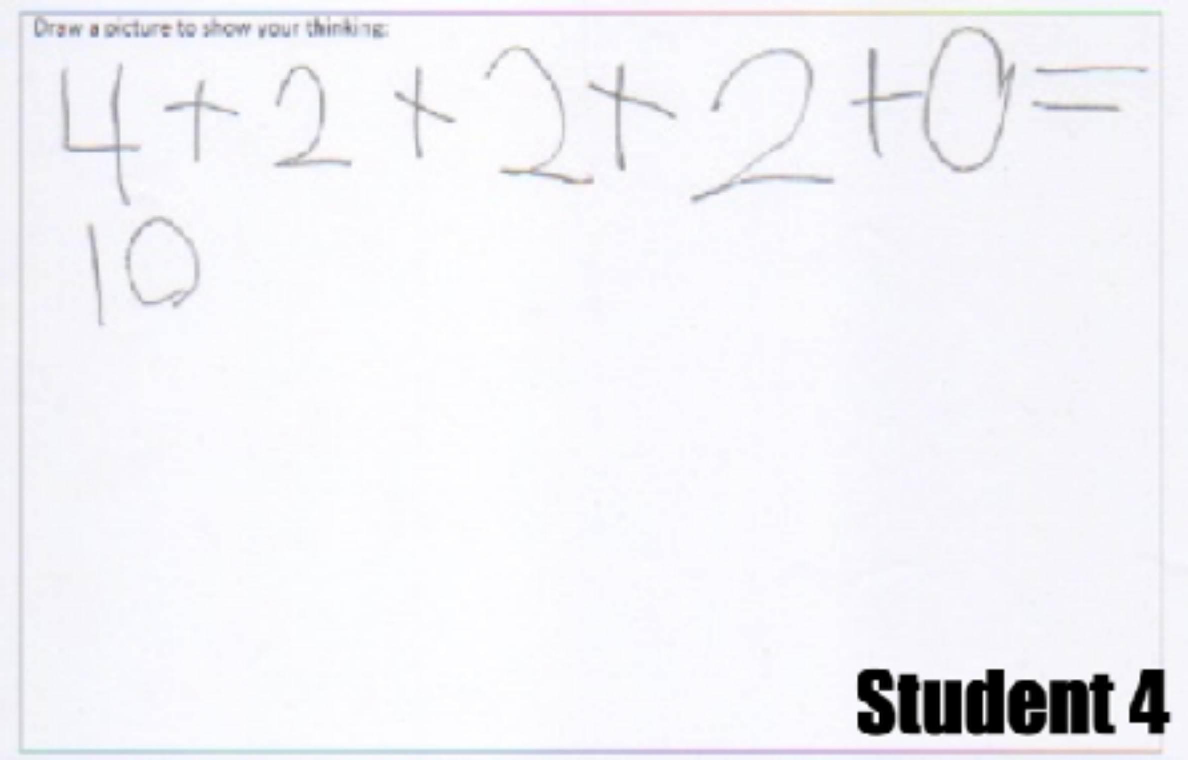

How would you sequence or connect these student work samples?

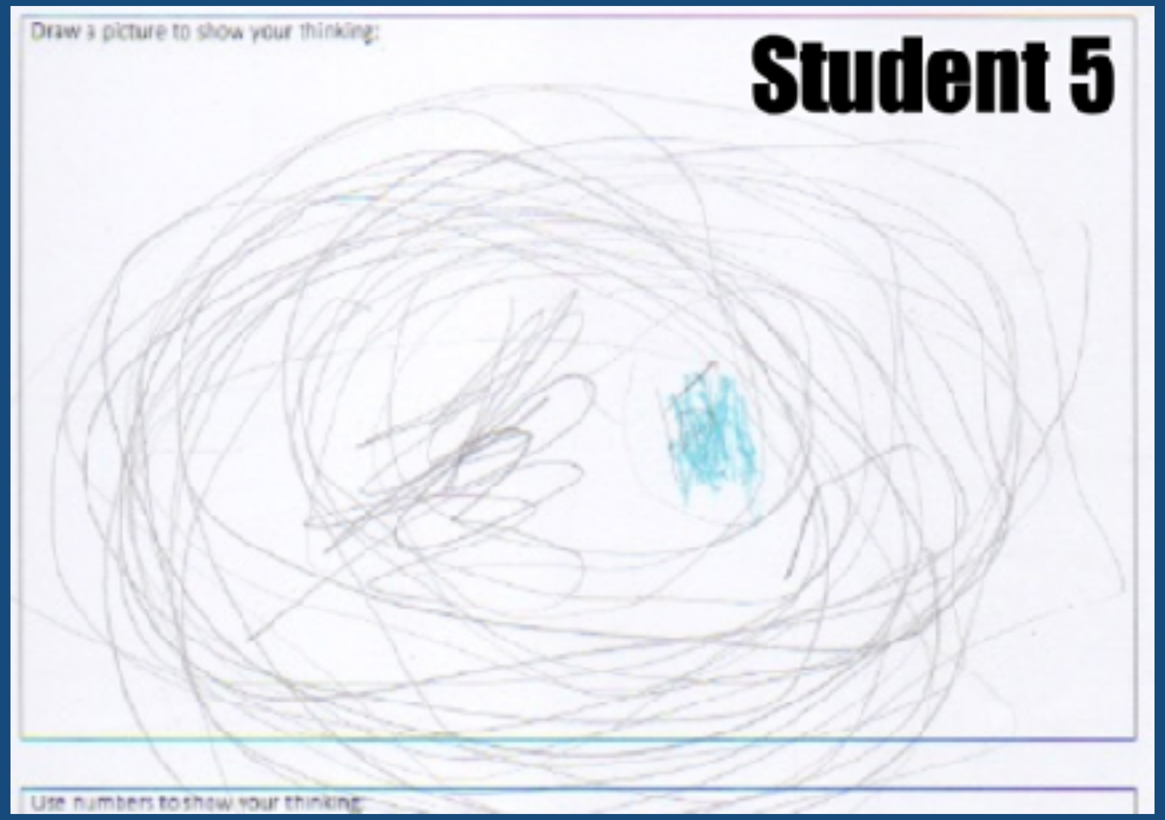
# Pause

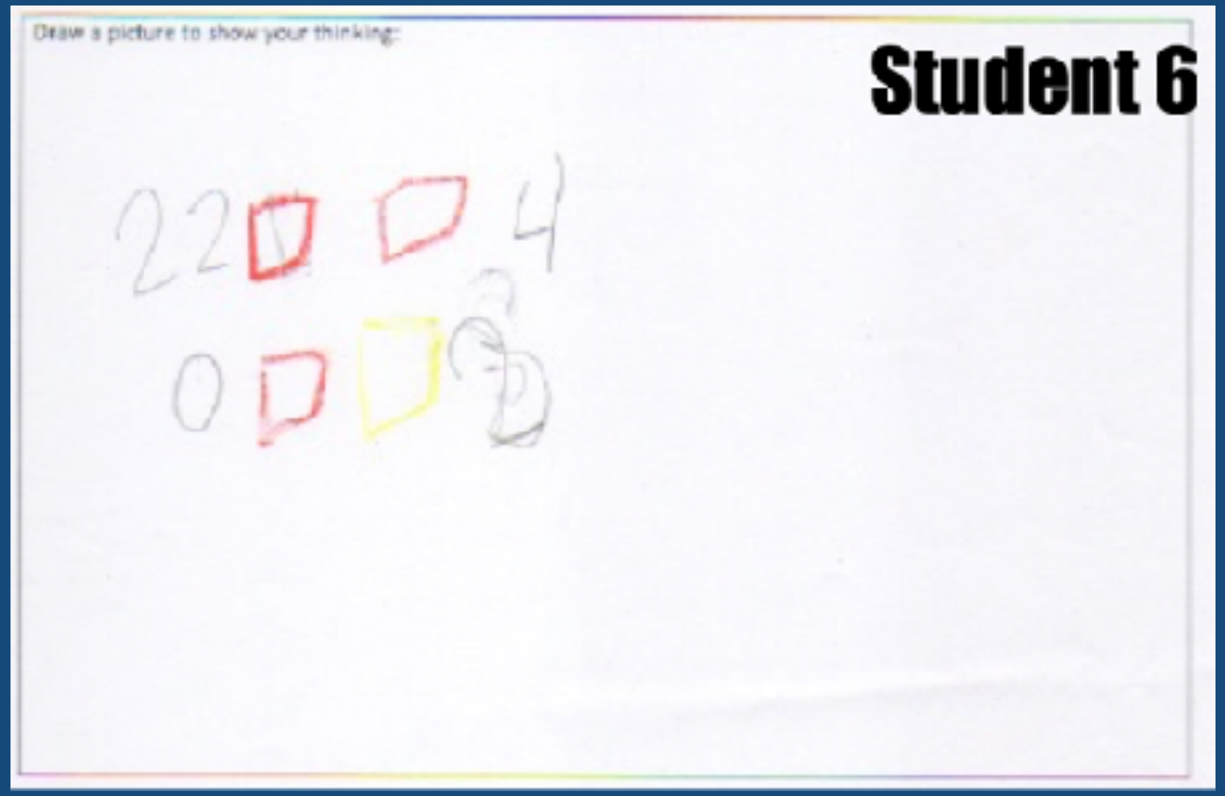


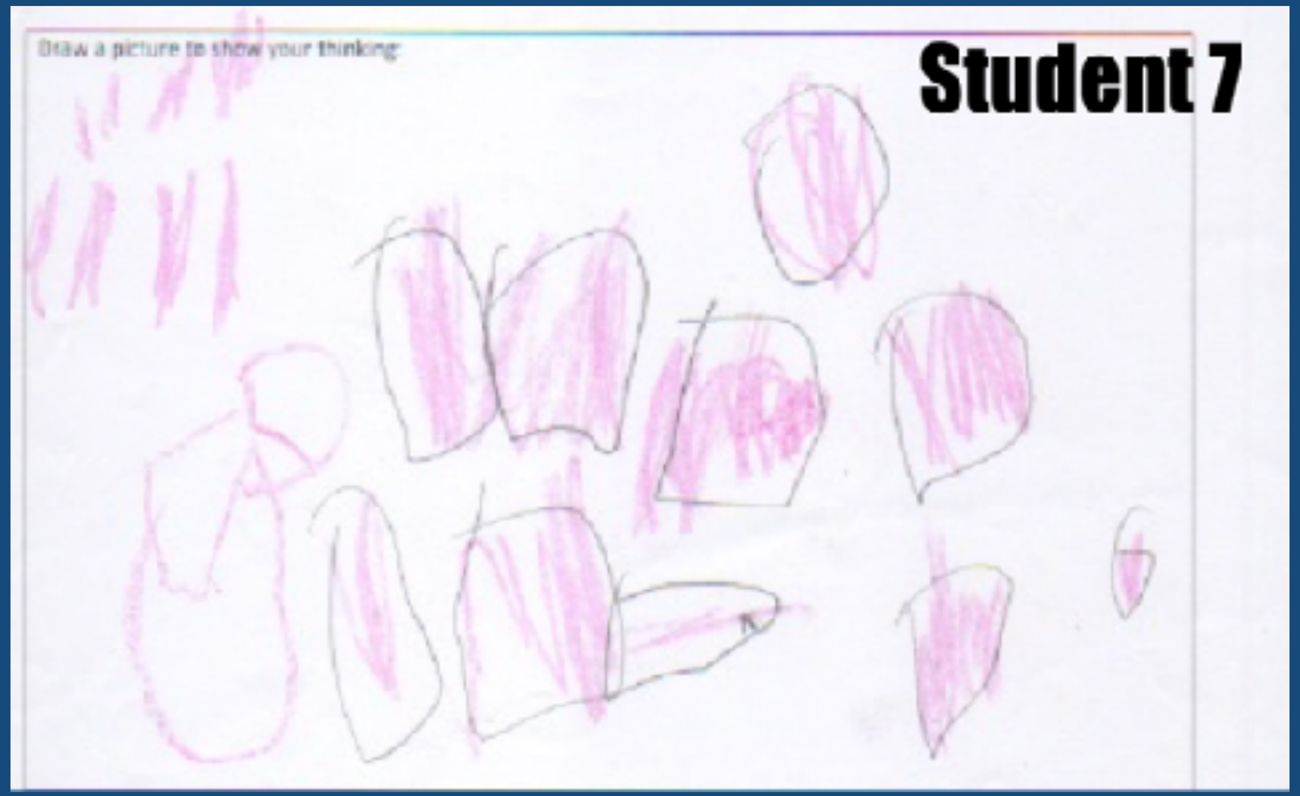



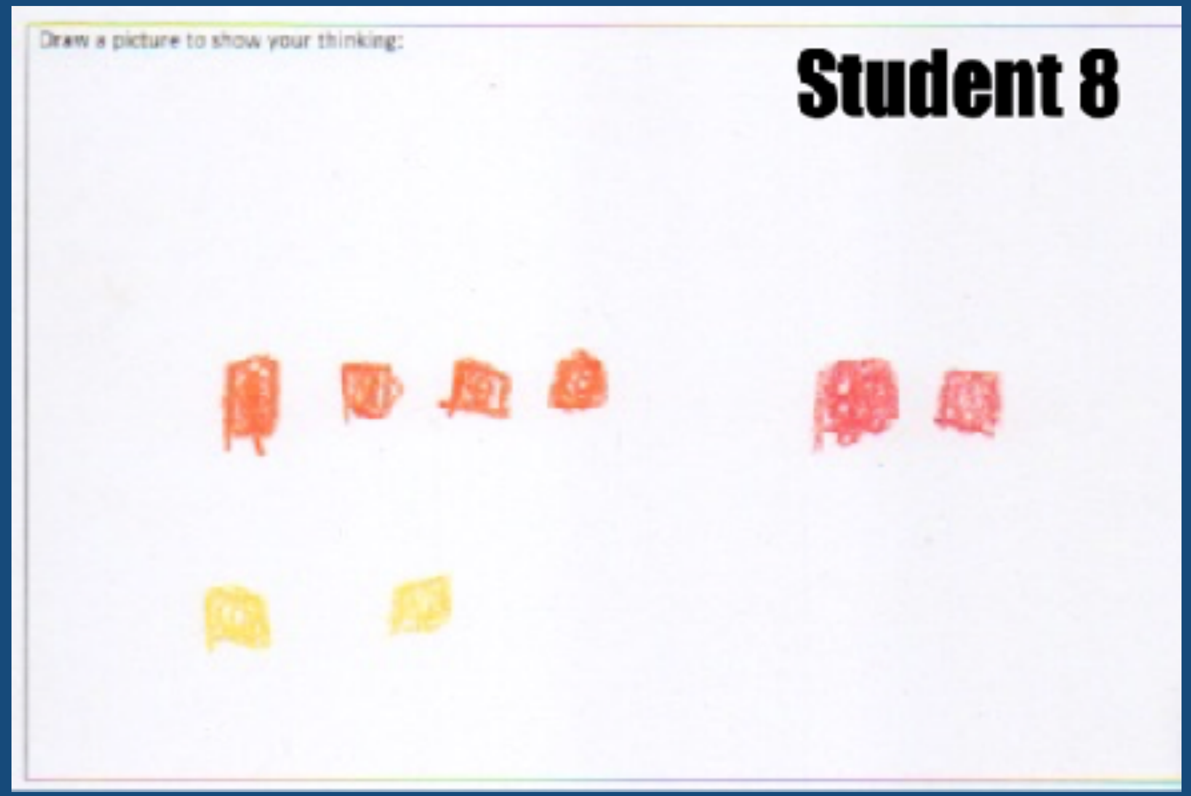

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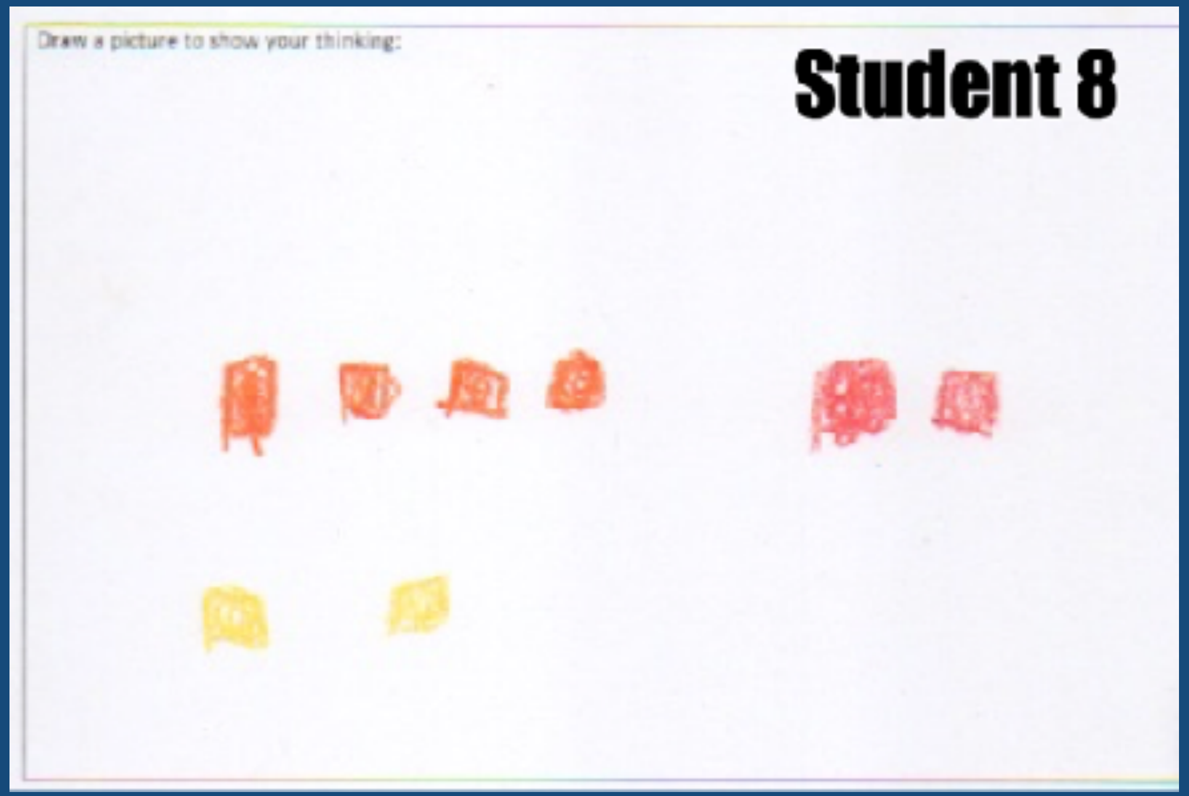
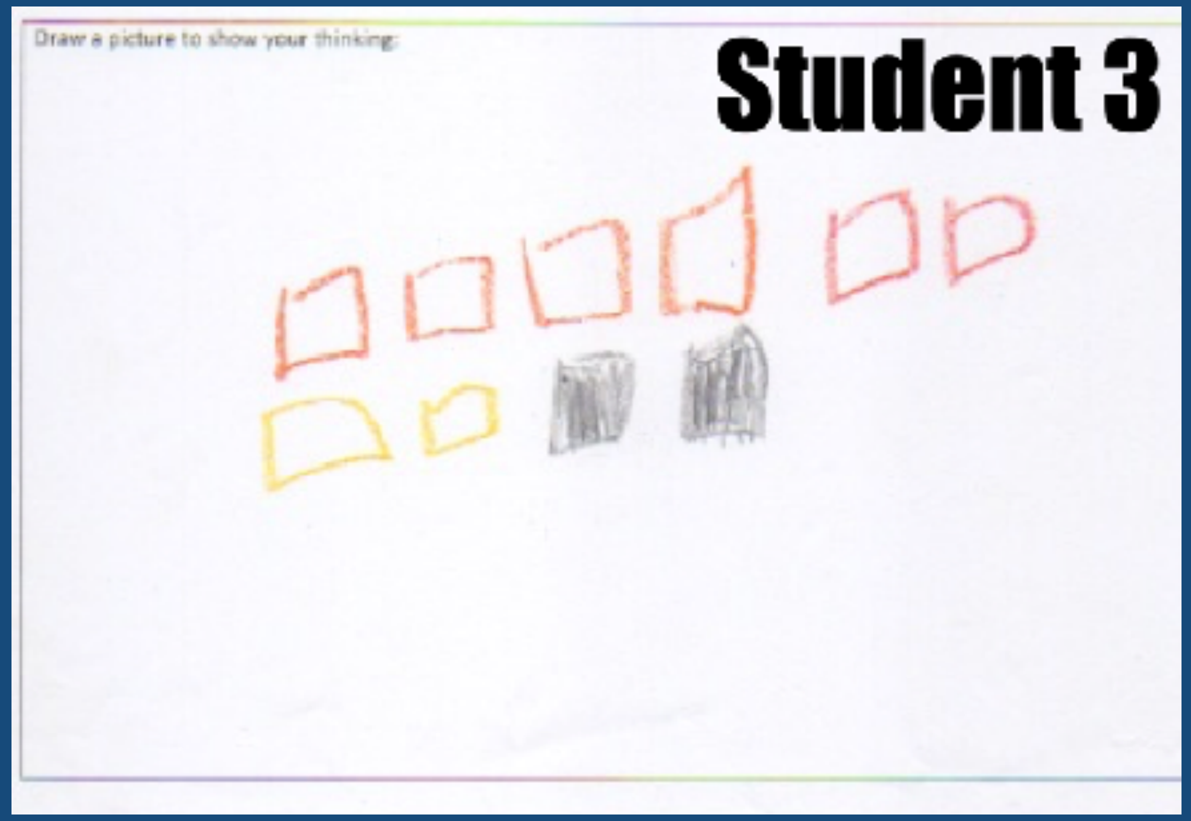
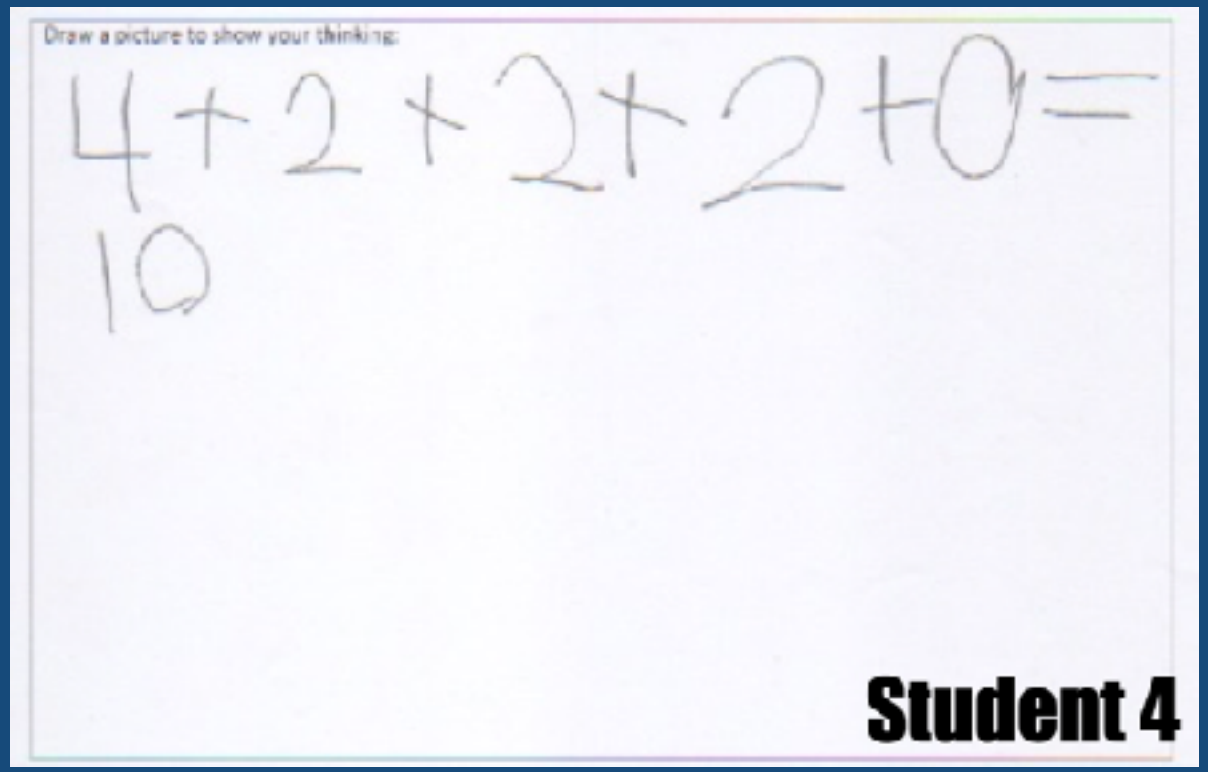
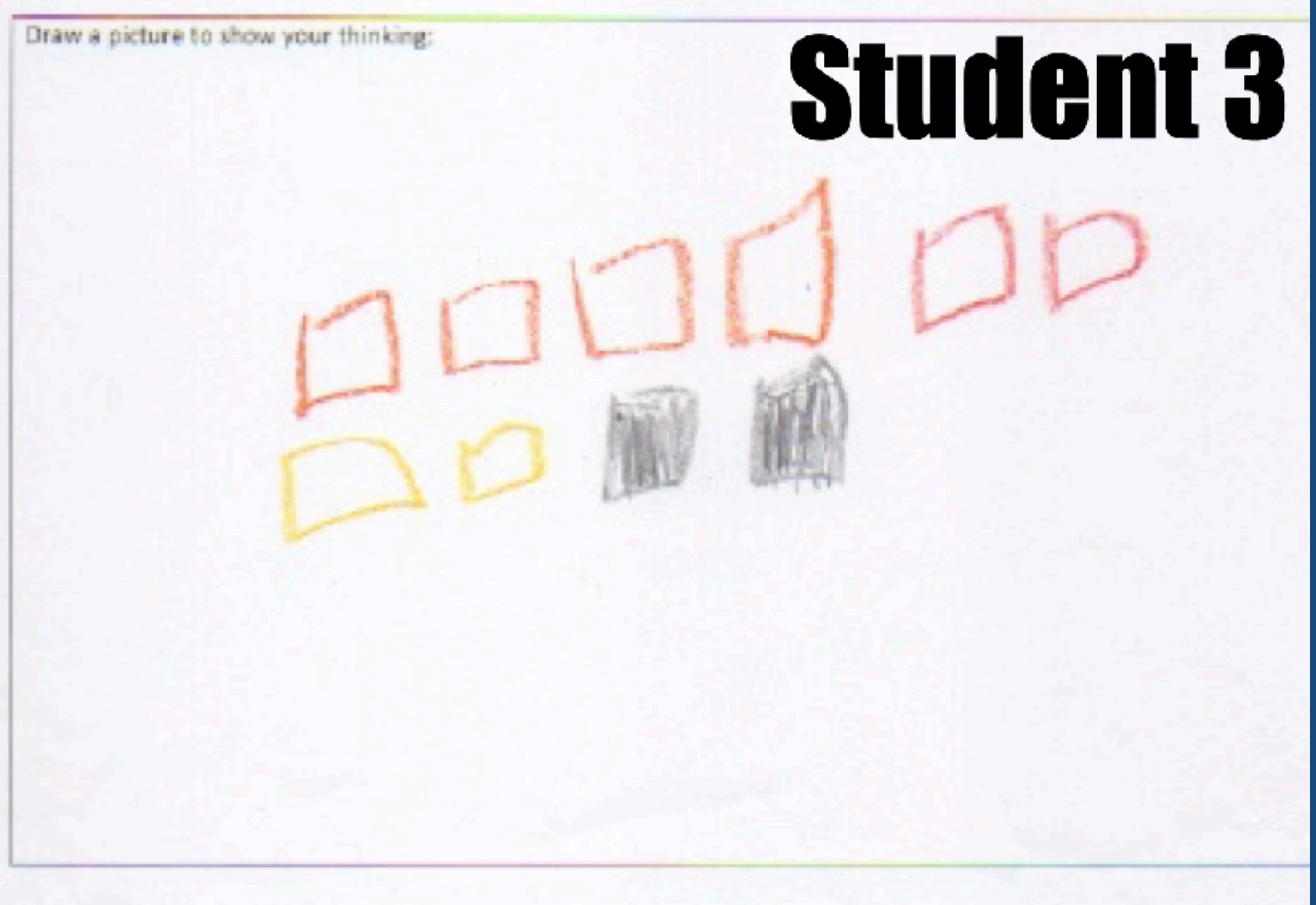
# Sequence and Connect

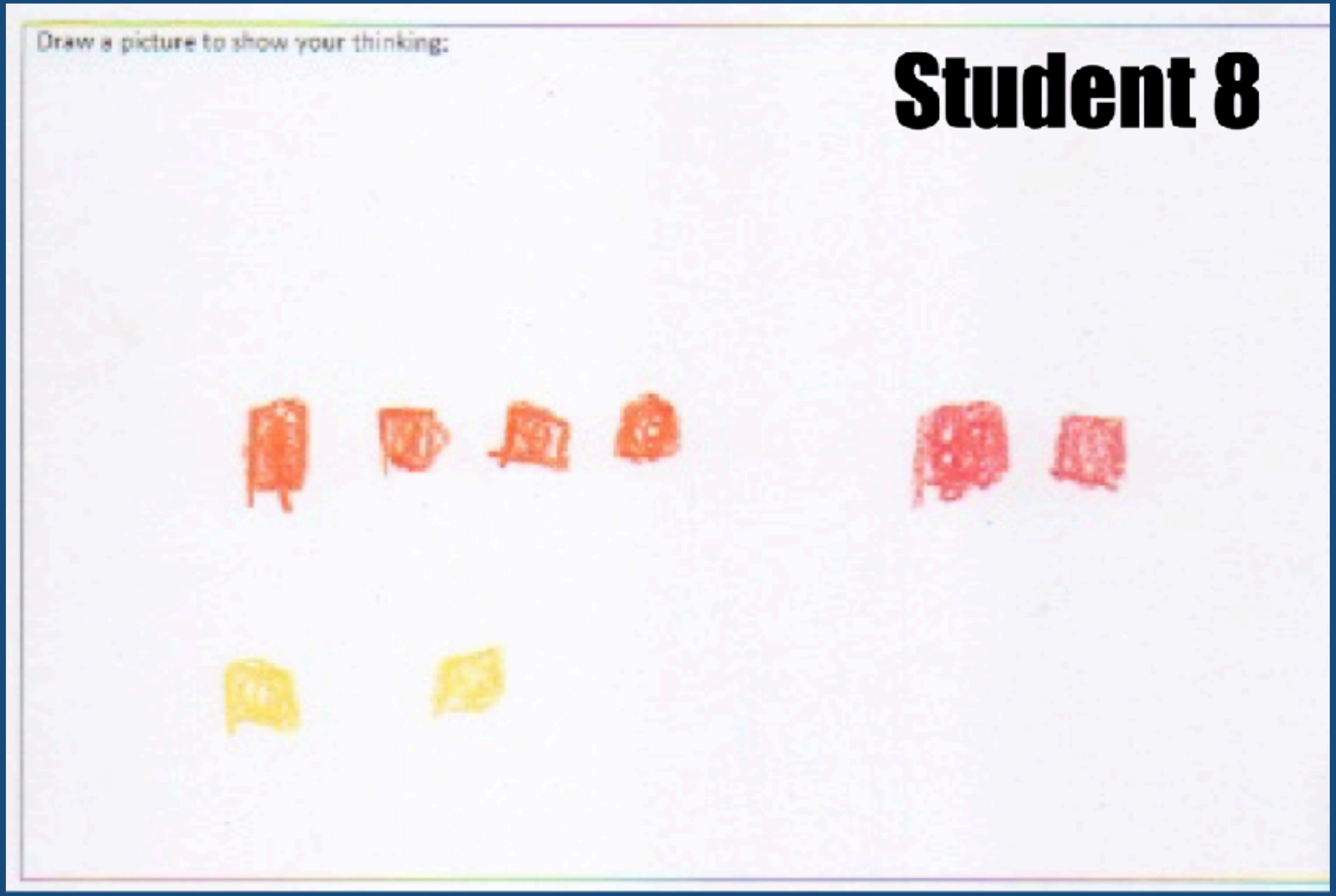


# Same? Different?



# What we learned...

- Has number recognition
- Number order and sequence
- Cardinality
- One to one correspondence
- Rote count
- Model sets
- Combine or join sets

# Module 1

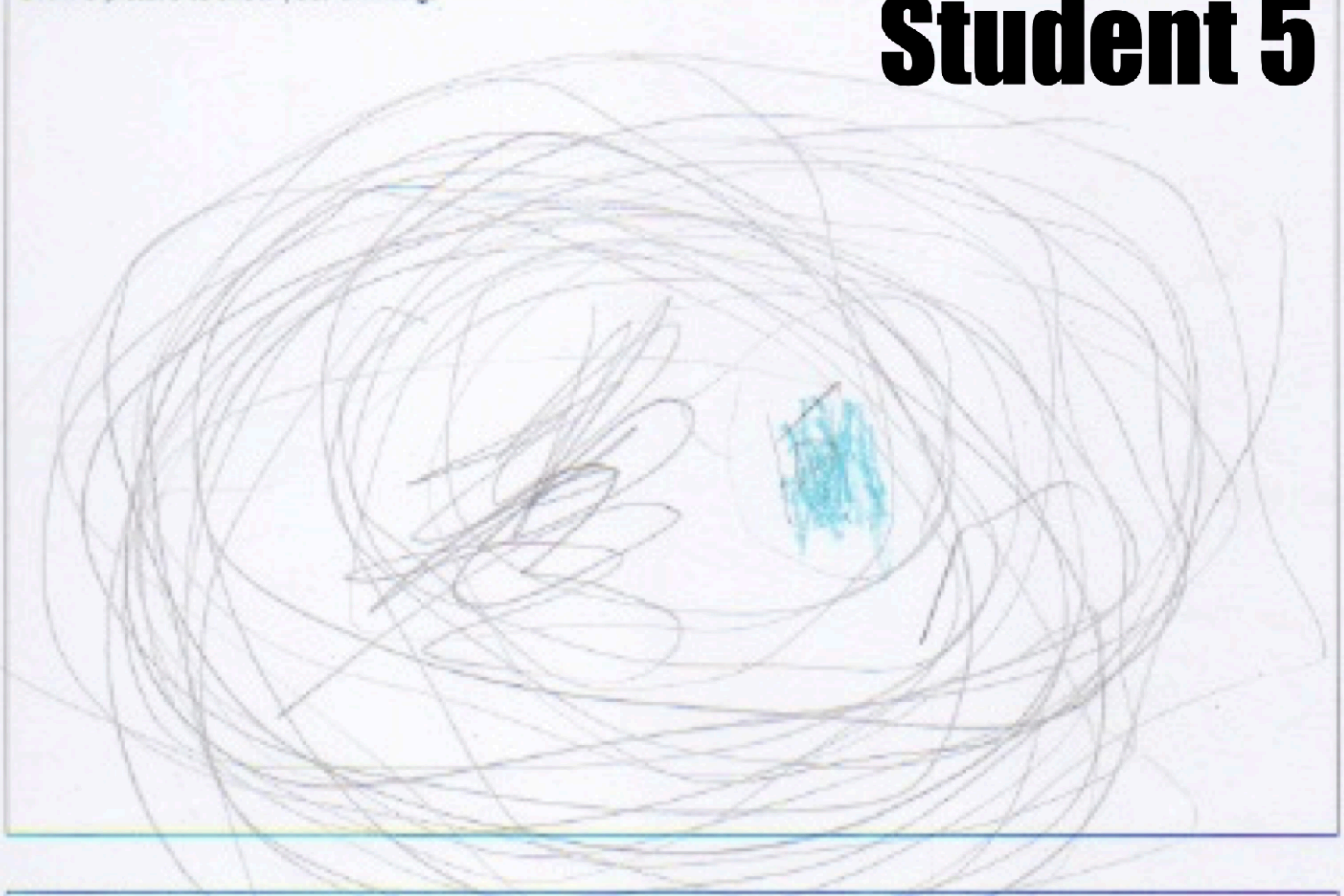
**Part 1:** Launch the Concept

**Part 2:** Connect and Extend Thinking

**Part 3:** Building Fluency

# Student 5

Draw a picture to show your thinking:



Use numbers to show your thinking:

# Where do we start?

# 2 Questions

# 1 Billion Circles

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# 1 Billion Circles

100 circles : minute

144,000 circles : day

1,000,000,000 would take 6944 days

19+ years with no sleep

?

# Where does 1 billion go on the number line?

# Where does 1 billion go on the number line?

| | |
|---|---|
| **One-to-One Correspondence** | Students can connect one number with one object and then count them with understanding. |
| **Counting** | The number of objects remains the same when they are rearranged spatially. 5 is 4&1 OR 3&2. |
| **Number Conservation** | Rote procedure of counting. The meaning attached to counting is developed through one-to-one correspondence. |
| **Hierarchical Inclusion** | Numbers are nested inside of each other and that the number grows by one each count. 9 is inside 10 or 10 is the same as 9 + 1. |
| **Subitizing** | Understanding that the last number in your count sequence tells you how many are in the set you counted. |
| **Comparison** | Being able to visually recognize a quantity of 5 or less. |
| **Cardinality** | Being able to compare quantities by identifying which has more and which has less. |

# Number Sense Trajectory

1. Cut out the headers and descriptors.

2. Match each header to a the correct descriptor.

3. Place the headers and descriptors in the progression for how student build and learn number.

| | |
|---|---|
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# Number Sense Trajectory

Comparison

Counting

Hierarchical Inclusion

Subitizing

Number Conservation

Cardinality

1-to-1 Correspondence

# Number Sense Trajectory

Subitizing

Comparison

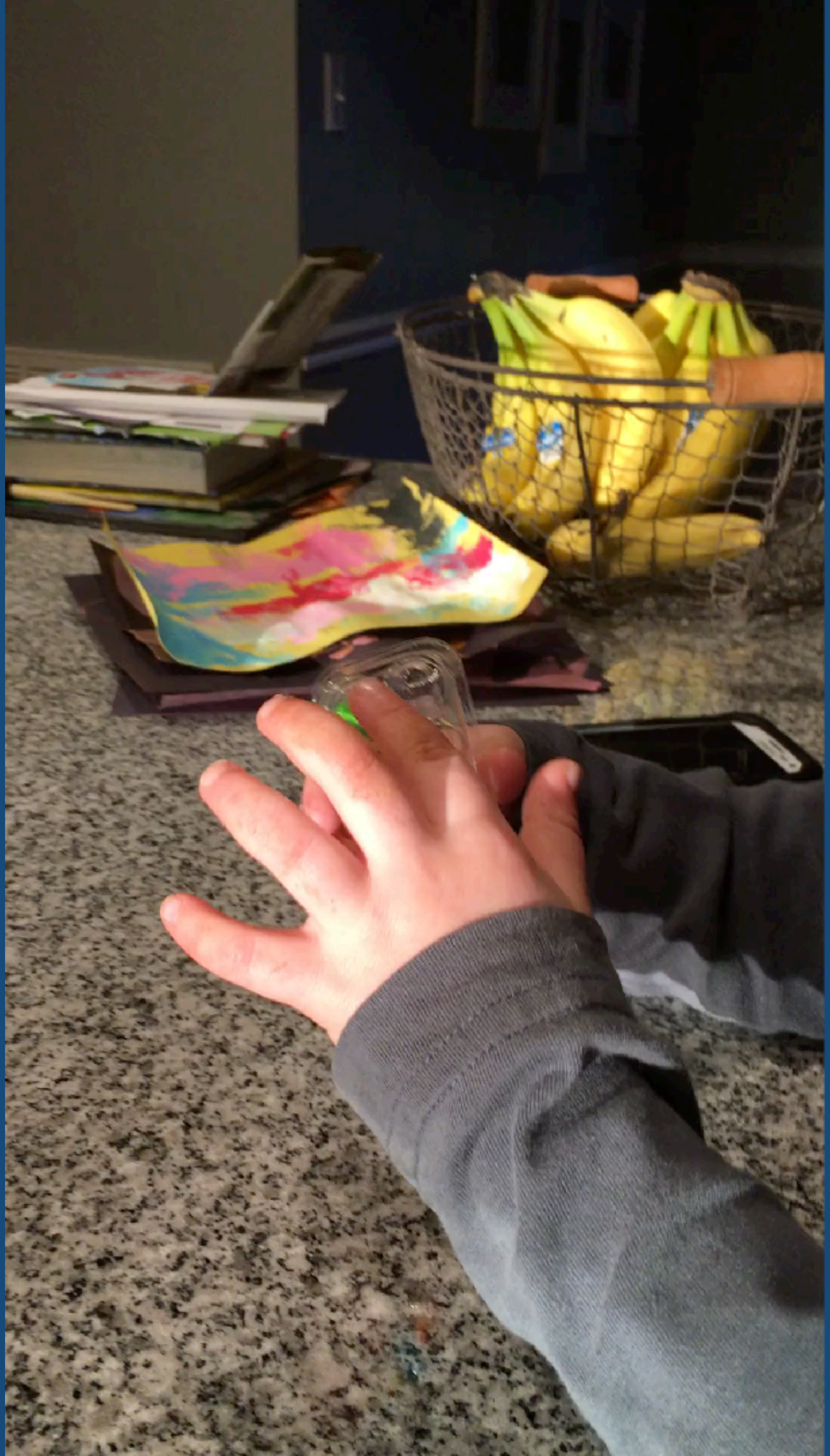
Counting

1-to-1 Correspondence

Cardinality

Hierarchical Inclusion

Number Conservation



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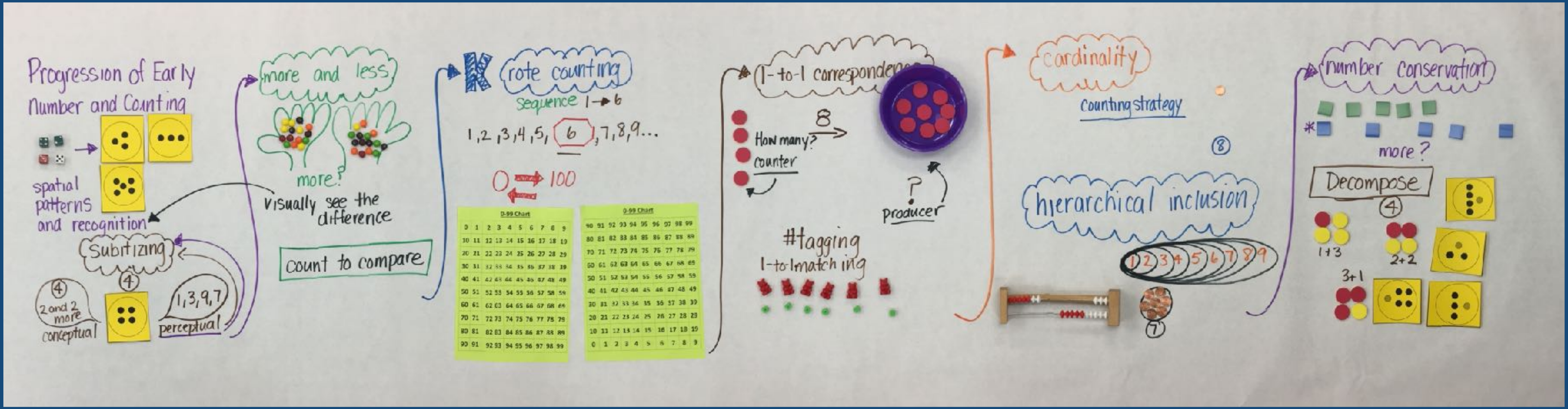
# Making Sense Series

the progression of early number & counting

created by Graham Fletcher
@gfletchy
www.gfletchy.com

# Progression of Early Number & Counting



# 5 Counting Principles

# Putting It All Together

| Trajectory | Subitizing & Comparison | Rote Counting<br>1-to-1 Correspondence<br>Cardinality | Hierarchical Inclusion | Number Conservation |
| --- | --- | --- | --- | --- |

5 Counting Principles

# 5 Counting Principles

**Stable Order:** Understanding the verbal sequence of counting; being able to say the number names in sequential order.

*Gelman, Rochel, and C. R. Gallistel. The Child's Understanding of Number Cambridge: Harvard University Press, 1978.*

# 5 Counting Principles

**One-to-One Correspondence:** Understanding that when saying the names of the numbers in sequence, each object receives one count and one only one count.

*Gelman, Rochel, and C. R. Gallistel. The Child's Understanding of Number Cambridge: Harvard University Press, 1978.*

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# 5 Counting Principles

**Cardinality:** Understanding that the last number spoken in a counting sequence names the quantity for that set.

*Gelman, Rochel, and C. R. Gallistel. The Child's Understanding of Number Cambridge: Harvard University Press, 1978.*

# 5 Counting Principles

**Abstraction:** Understanding that it doesn’t not matter what you count, how we count stays the same. For example, any set of objects can be counted as a set, regardless of whether they are the same color, shape, size, etc. This can also include non-physical things such as sounds, imaginary objects, etc.

*Gelman, Rochel, and C. R. Gallistel. The Child's Understanding of Number Cambridge: Harvard University Press, 1978.*

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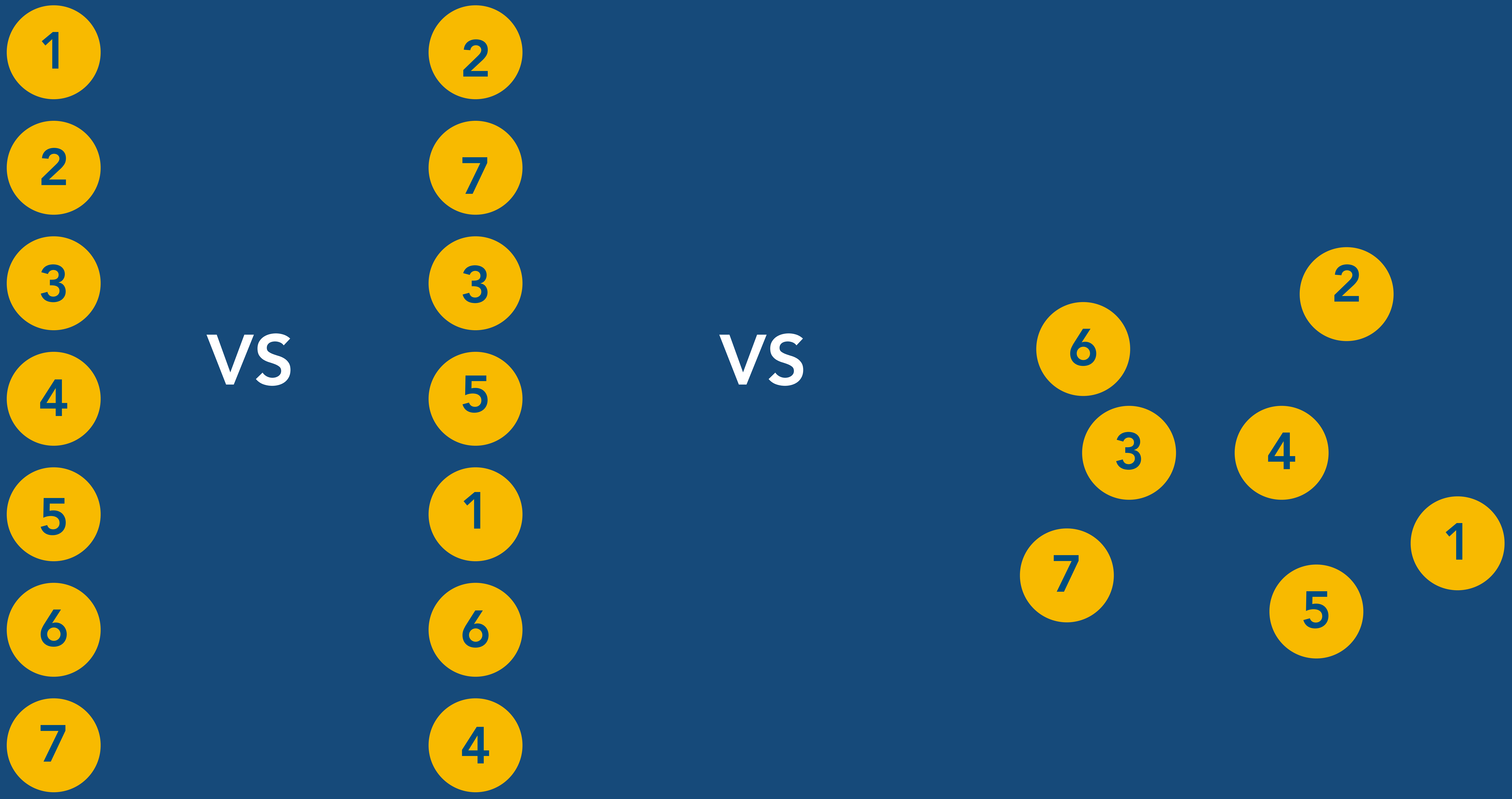
# 5 Counting Principles

**Order Irrelevance:** Knowledge that the order that items are counted in is irrelevant (left-to-right, right-to-left, in a random fashion) as long as every object in the set is given one count and only one count.

*Gelman, Rochel, and C. R. Gallistel. The Child's Understanding of Number Cambridge: Harvard University Press, 1978.*

# Order Irrelevance



# The Progression of Early Number and Counting

# Putting It All Together

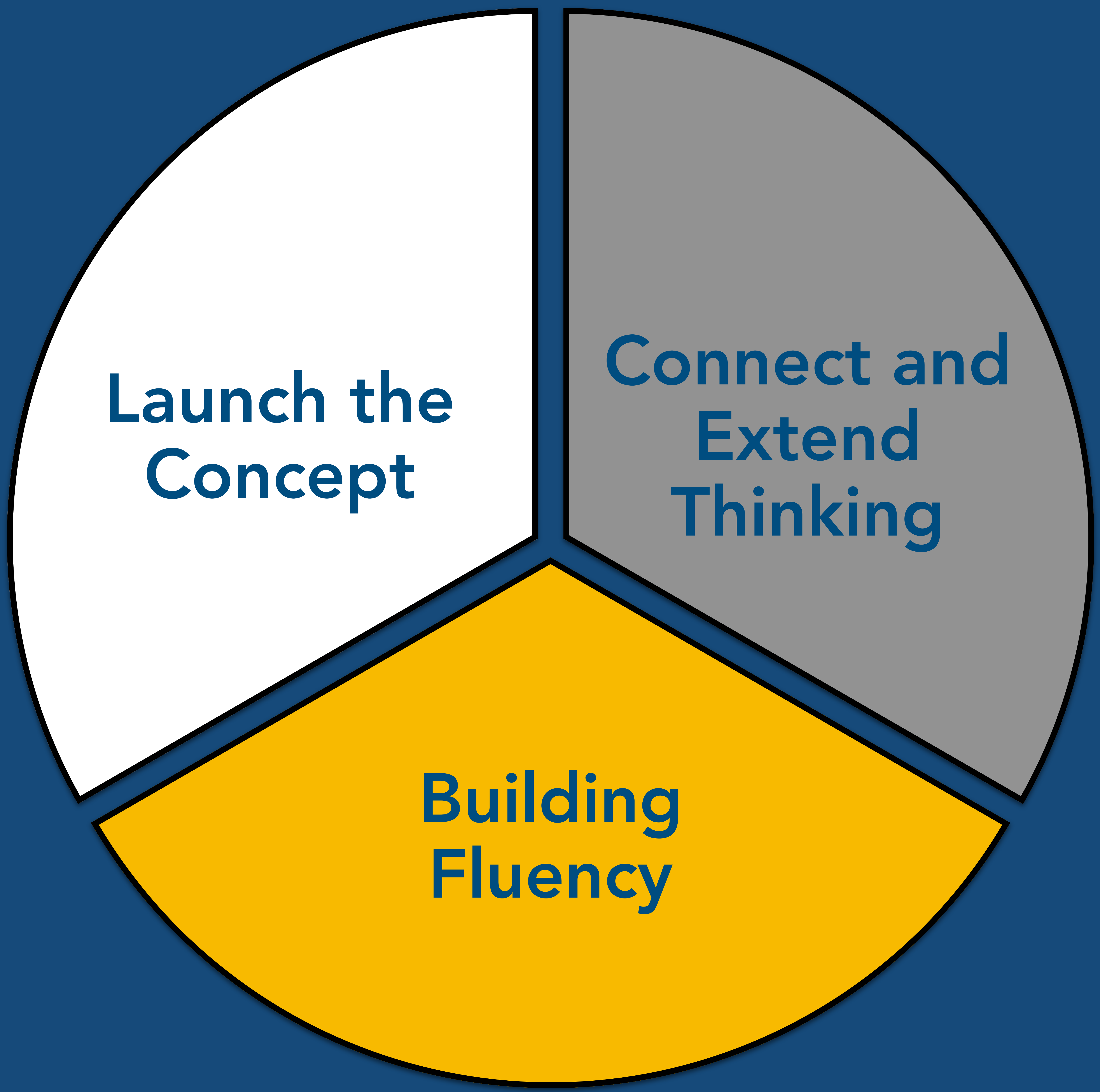
| | | | | |
|---|---|---|---|---|
| Trajectory | Subitizing & Comparison | Rote Counting<br>1-to-1 Correspondence<br>Cardinality | Hierarchical Inclusion | Number Conservation |
| Number Relationships | Spatial Relationships & Patterned Recognition | One & Two More & Less | Understanding Anchors | Part-Part-Whole & Unitizing |

# Module 1

**Part 1:** Launch the Concept

**Part 2:** Connect and Extend Thinking

**Part 3:** Building Fluency


Launch the Concept
Connect and Extend Thinking
Building Fluency

# Building Fluency

Number Sense Trajectory Games
5-10-20 Frames

# Building Fluency

Rekenrek

# Building Fluency

Geometric Subitizing Cards

# Coming Up...

**Week 1:** Early Number and Counting
**Week 2:** Building Fluency through 20
**Week 3:** Building Fluency through 120
**Week 4:** Building Fluency through 1000
**Week 5:** Number and Measurement
**Week 6:** Number and Statistical Reasoning

# Foundations of Numerical Reasoning in Grades K-2

## Module 1: Early Number and Counting

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