Bridging Whole Number to Fractions with Mental Actions

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About Me:

In education for 13 years

- **4th Grade Teacher**
- **Math Instructional Coach**
- **Math Specialist**
- **Learning and Design Specialist, US Math Recovery Council**
- **2018 PAEMST (Presidential Award of Excellence in Math Teaching) Awardee**

Learning Goals

- Connect how whole number instruction support fractions instruction
- Identify how mental actions support conceptual understanding of fractions

4/5 + 11/12 is about

a.1

b.2

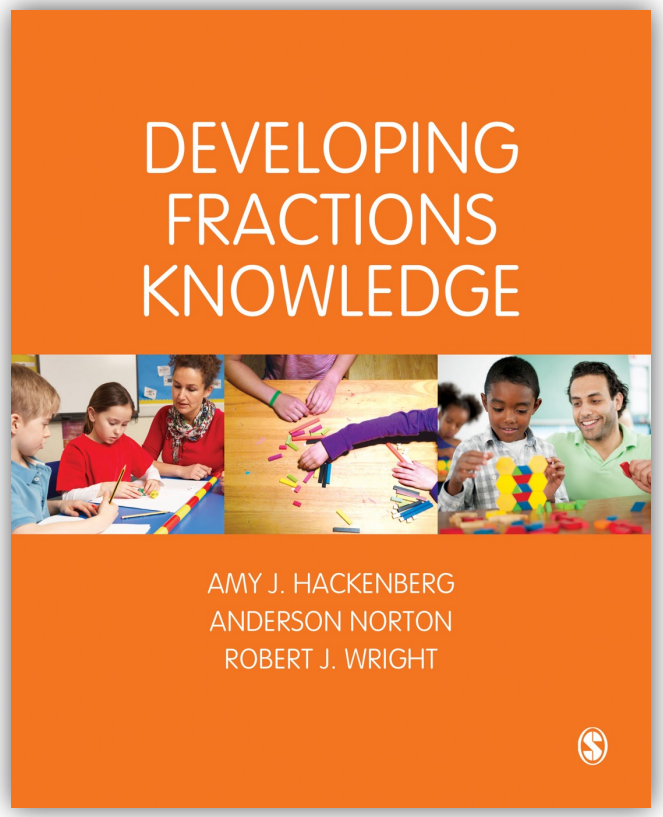
c.15

d.17

Why are Fractions difficult for students?

- Curricular materials are focused on the part/whole world
- Students are often asked to memorize rules for fractions with no conceptual understanding
- Rules that work for whole number don’t work anymore for fractions

Developing Fractions Knowledge

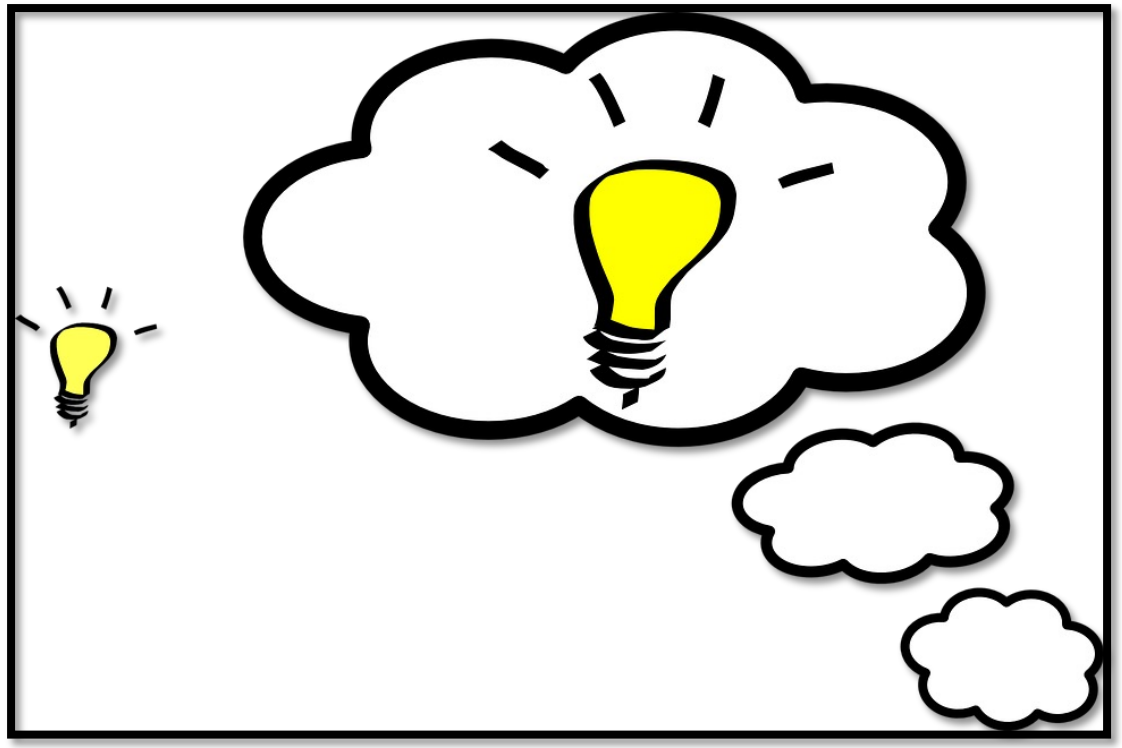


Dr. Bob Wright
Southern Cross University

Dr. Amy J. Hackenberg
Indiana University

Dr. Anderson Norton
Virginia Tech University

Interference Hypothesis



- Students whole number knowledge can interfere with their development of fractions knowledge
- How could whole number knowledge interfere with students comparing fractions?

Reorganization Hypothesis

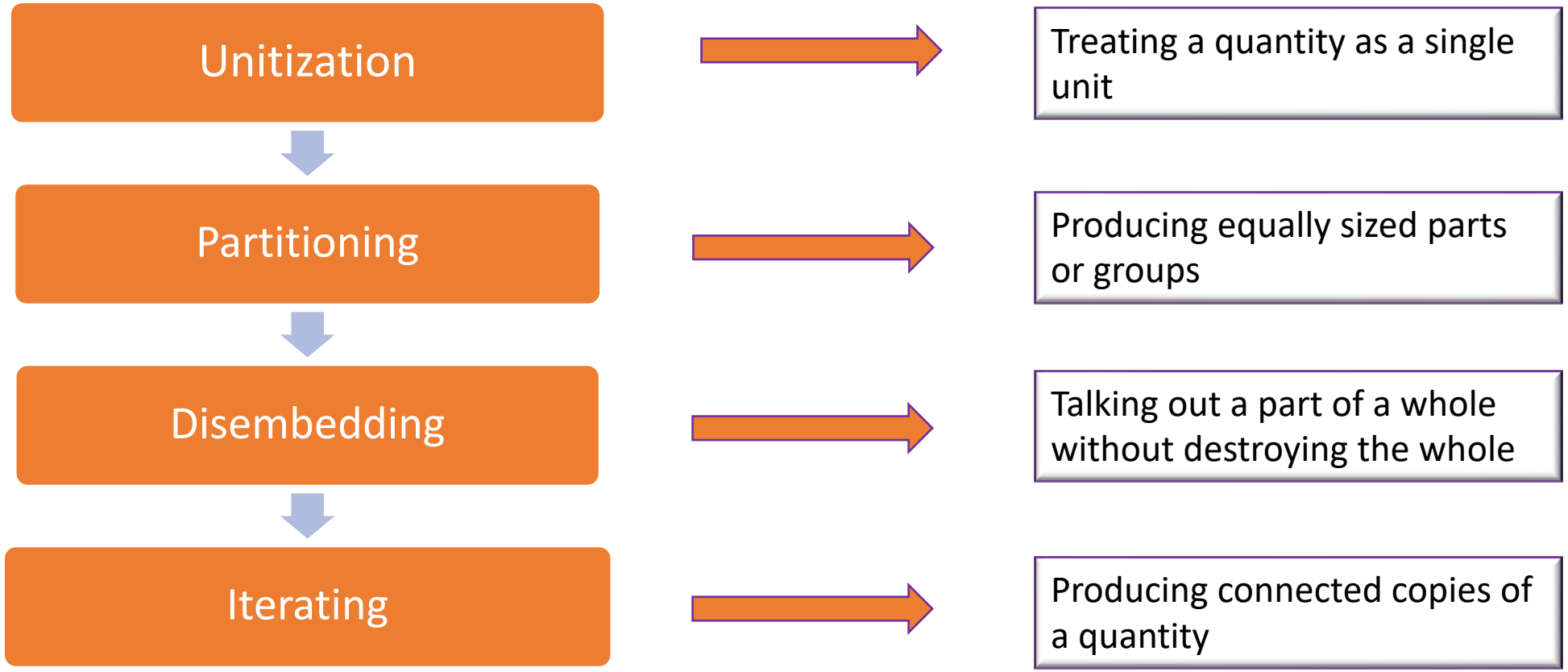
- The way students work with whole number influences the way they engage with fractions instruction
- Students' whole number knowledge can interfere with their development of fractions knowledge

What are Mental Actions?

- Supports students' ways of conceptually working with whole number and fractions

"Students perform many of the same mental actions to make sense of fractions that they perform to make sense of whole numbers."
-Hackenberg, Norton, Wright (Developing Fractions Knowledge, p. 16)

Trajectory of Mental Actions: Part/Whole to Measurement

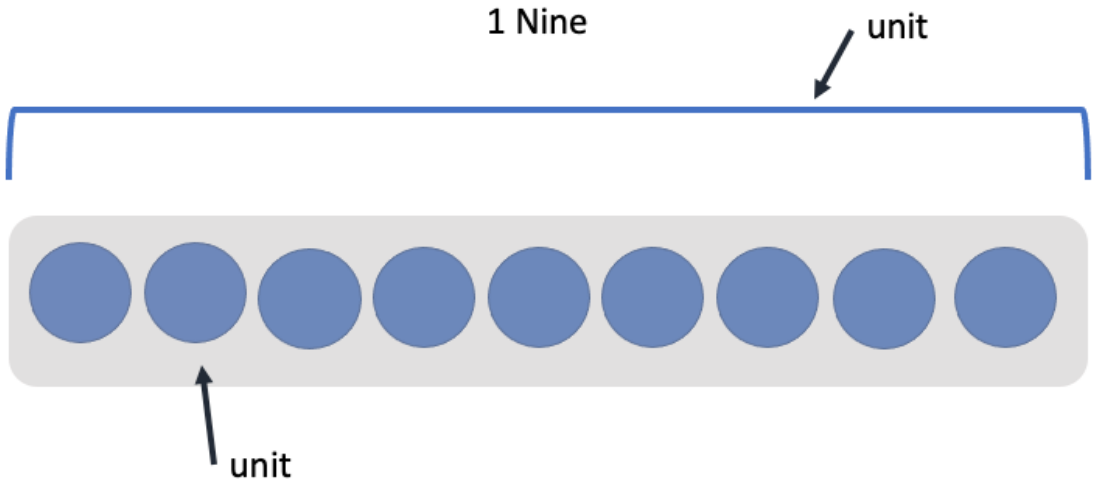


Mental Actions and the Common Core State Standards

- 4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Mental Action: Unitization

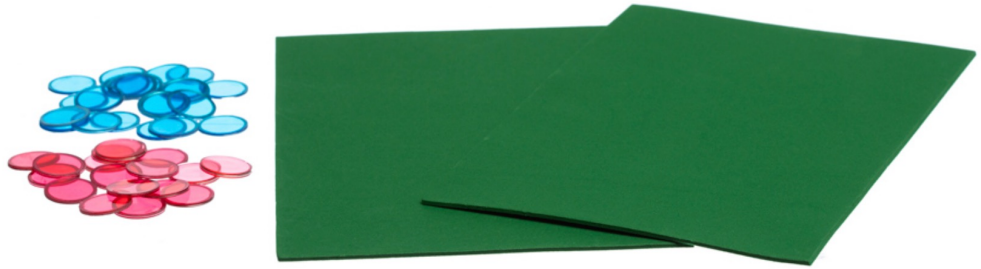
- Thinking of a collection of units as a whole
- Arises through student operation of counting and working with whole number



Unitization: Whole Number

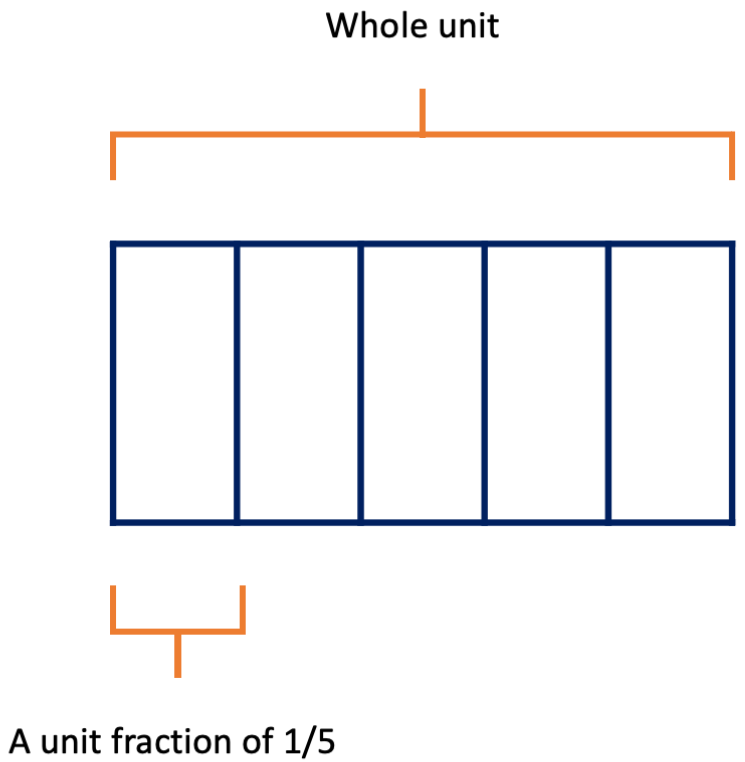
Counting: Developing an Initial Composite

- Count the collection for a number to exist
- Students unitize to create a composite to build from



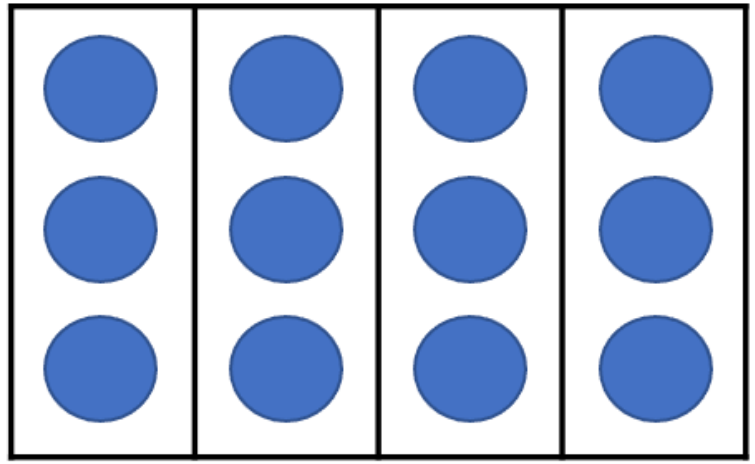
Unitization: Fractions

- A whole, unit of 1, can be regarded as a composite units made of smaller units
- For example, a whole can be thought of as containing five 1/5 units



Mental Action: Partitioning

- Creating equal size parts or groups while exhausting the whole



Partitioning: Whole Number

Domain: Multiplication and Division

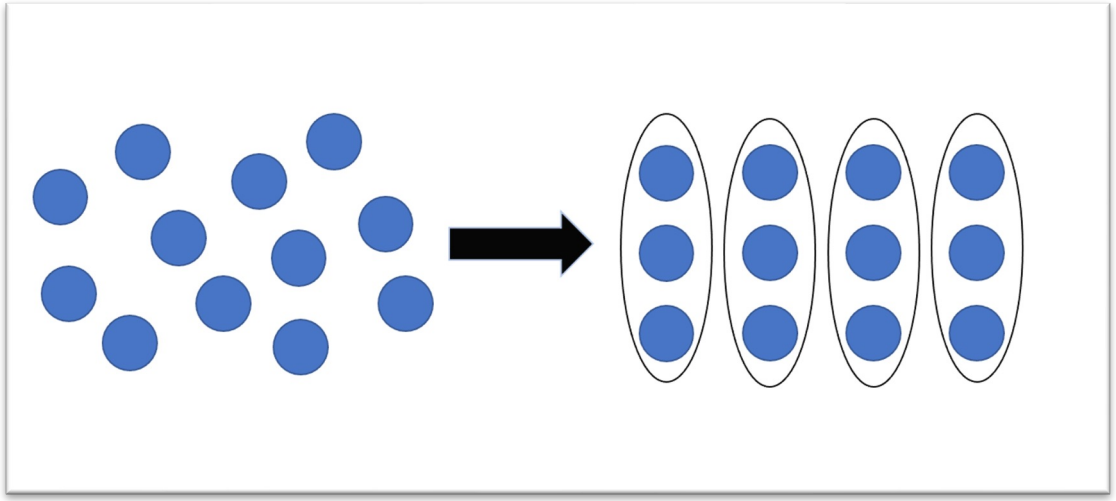
Topic: Forming Groups of a Specified Size

Setting: Counters

What to say: "Can you share the counters into groups of 3?

Partitioning: Whole Number

Can you share the counters into groups of 3?

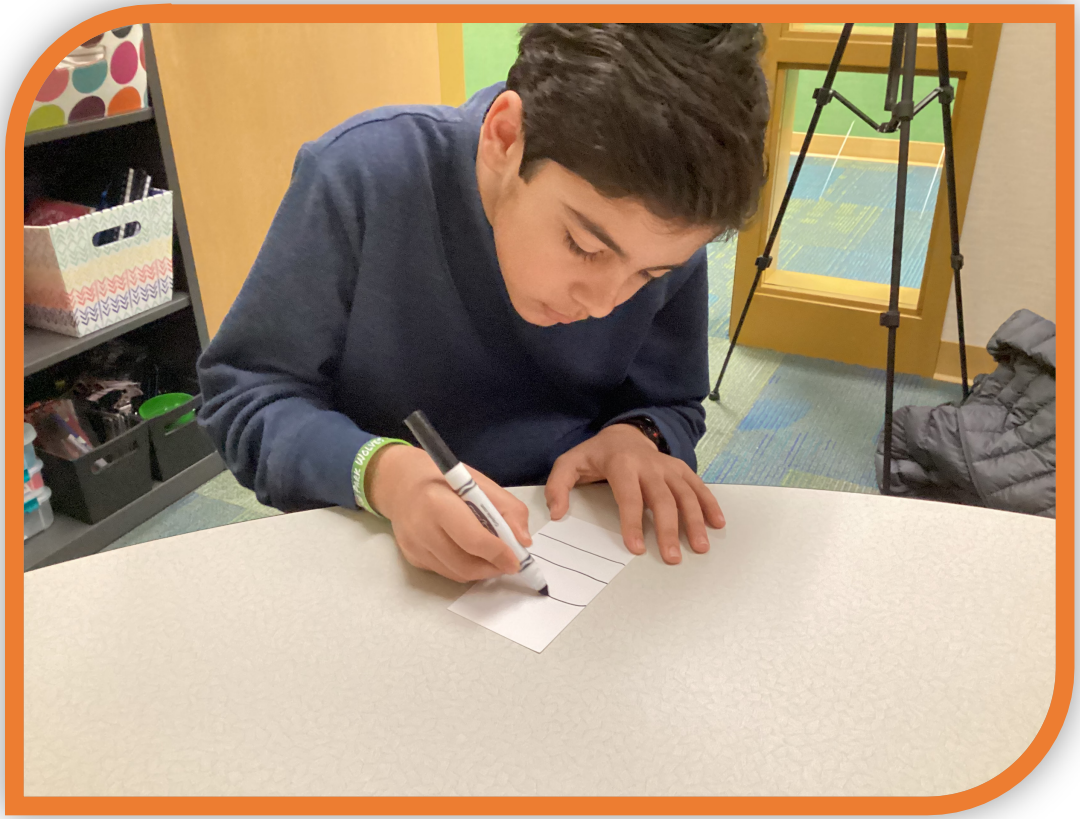


Operation (Mental Action)

- Partitioning a collection of objects
- Equally sharing a "collection of objects
- Students may see the 4 units of 3 within 12

Partitioning: Fractions

- Projecting a composite unit into a continuous whole
- Requires the coordination of three important aspects
  - Making appropriate number of shares
  - Creating equal-sized shares
  - Exhausting the whole



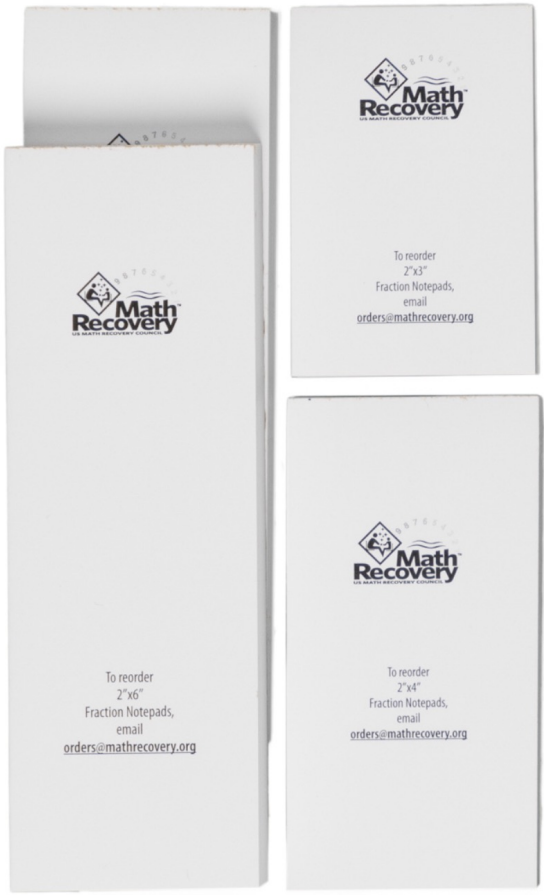
Partitioning: Fractions

Domain: Fractions

Topic: Parts Within Wholes

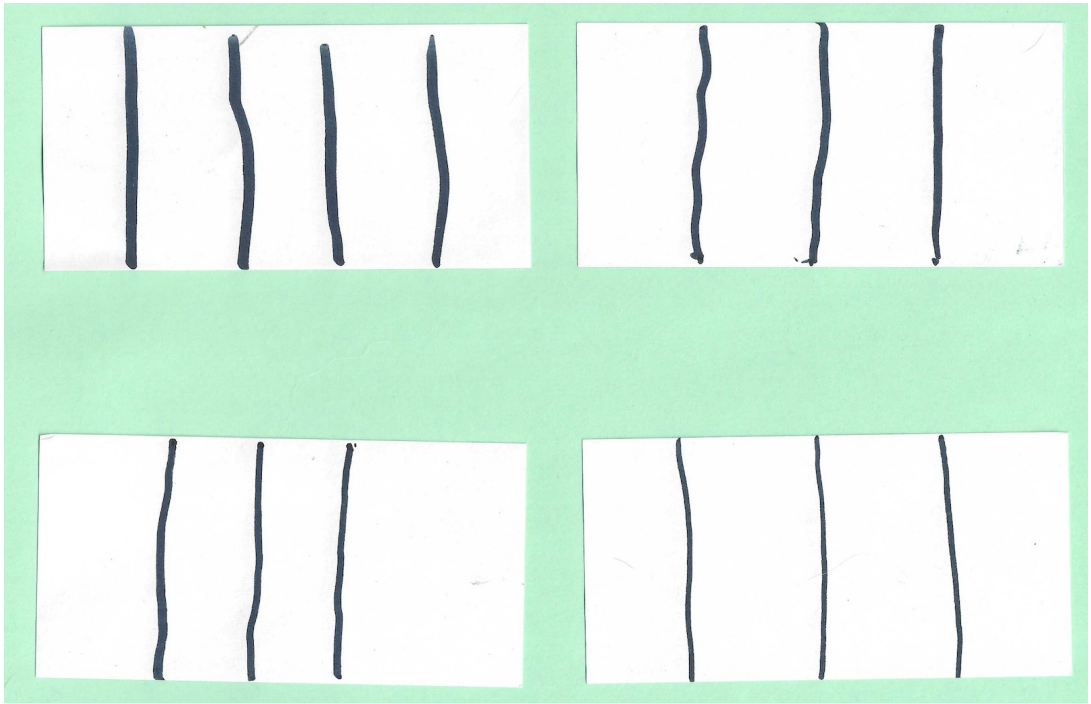
Setting: Various Rectangles

What to say: “Share a candy bar among four friends. Show each share.”



Partitioning: Fractions

“Share a candy bar among four friends. Show each share.”



Operation (Mental Action)

- Produce an equal size parts, appropriate number of shares, and exhausting the whole

How do you support the mental action of partitioning in fractions?

Questioning

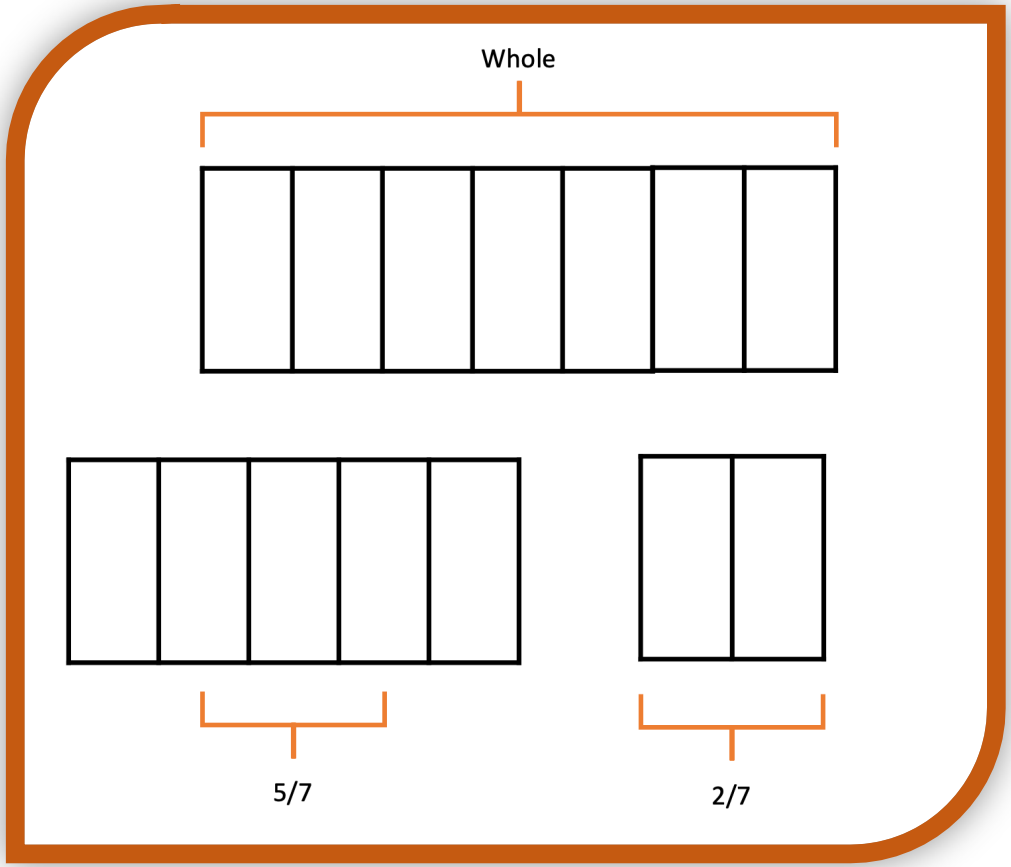
- How did you decided where to make the marks?
- If all the parts are not equal, how would you make them equal?
- Is there a share that would be an equal share for ____ people?

Task Selection

- Number of Shares
- Sharing v. Fraction Language
- Paper v. Cardboard Rectangles

Mental Actions: Disembedding

- Taking out a part of a whole without destroying the whole



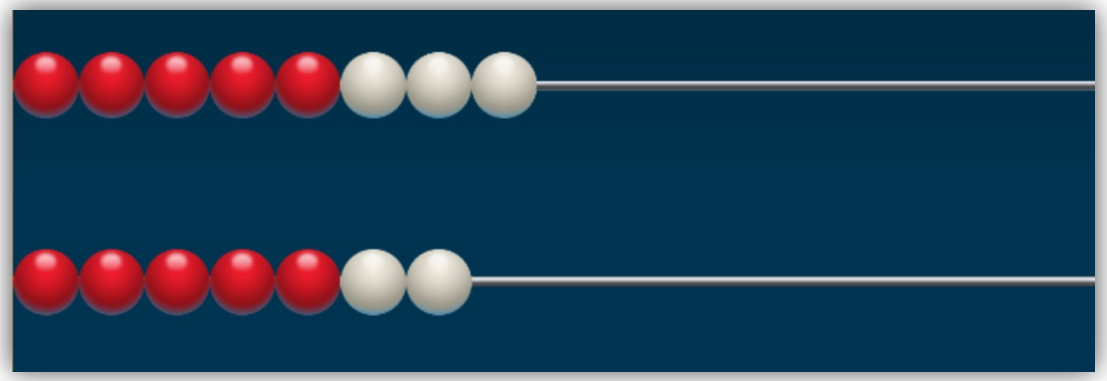
Disembedding: Whole Number

Domain: Addition and Subtraction to 20

Topic: Structuring Numbers 1-20

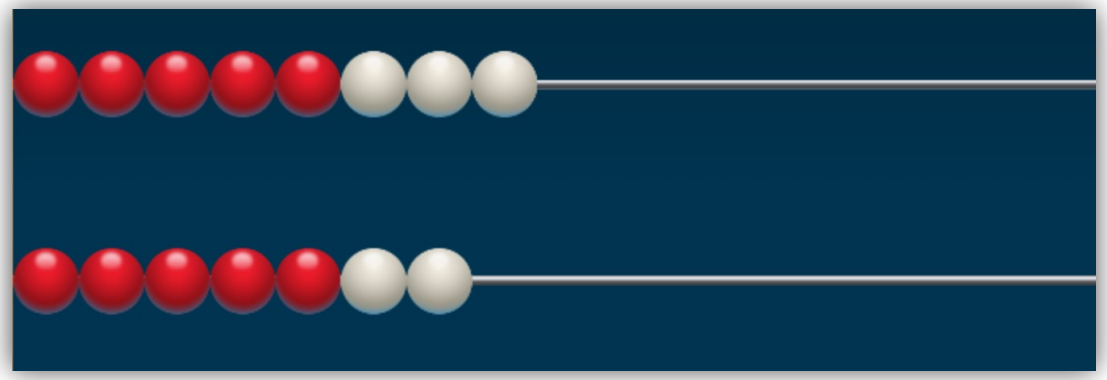
Setting: Bead Rack

What to say: "What is 8 + 7?"



Disembedding: Whole Number

What is 8 + 7?

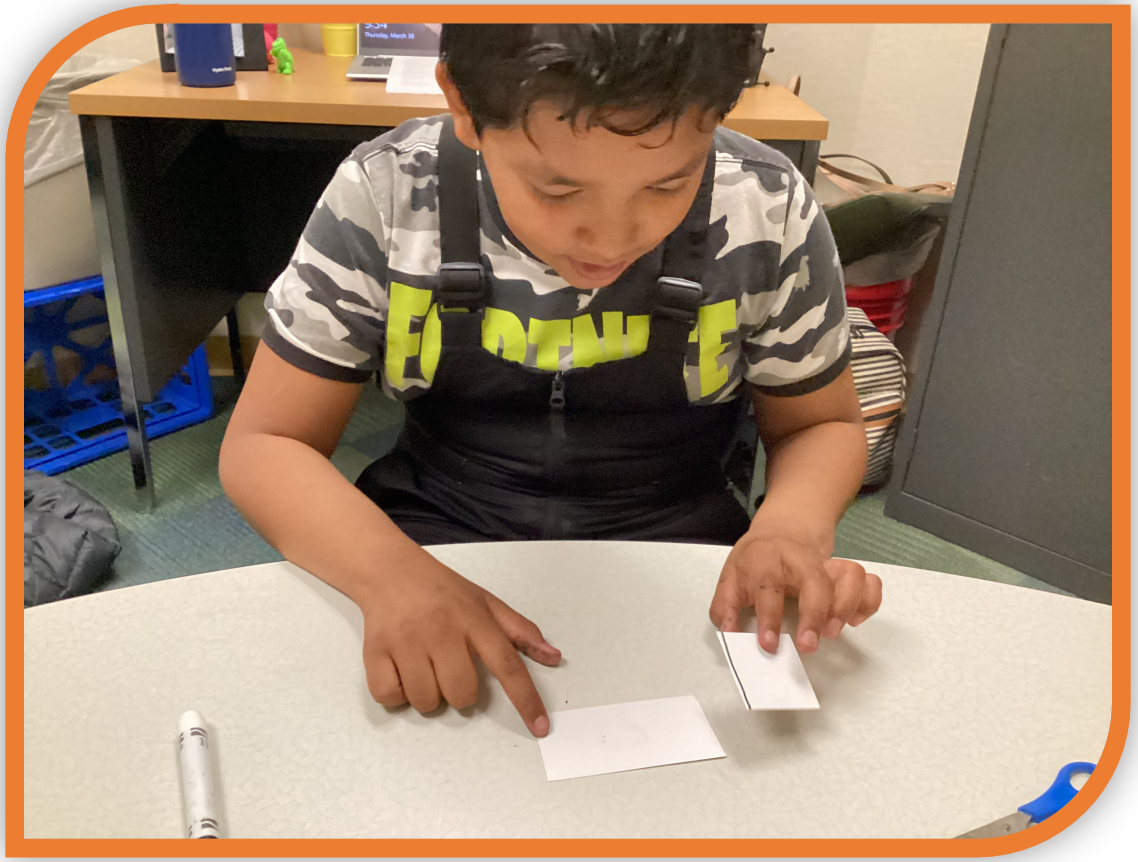


Operation (Mental Action)

- Students may disembed the 2 from the 7 to add through ten.
- Students may disembed the 7 from 8 to add using a double

Disembedding: Fractions

- Taking out a part of a whole without destroying the whole
- Fractions become known as a “size” amount



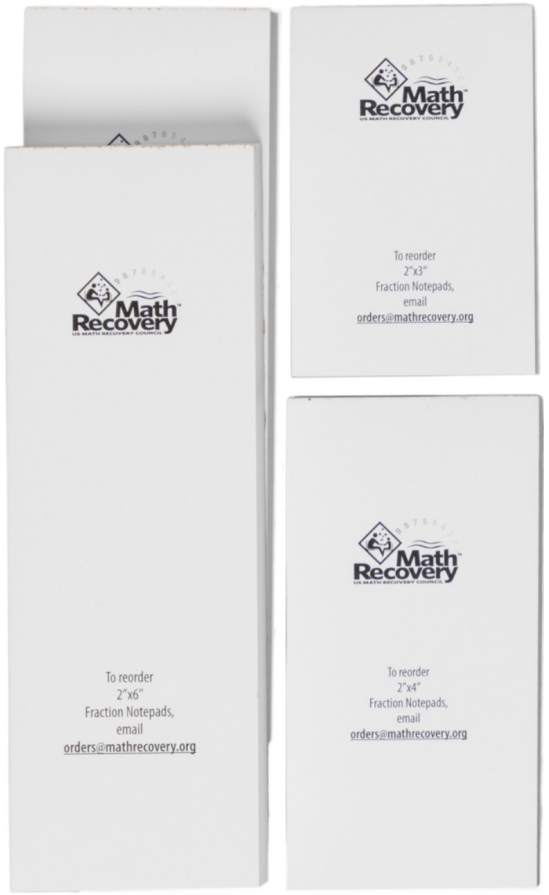
Disembedding: Fractions

Domain: Fractions

Topic: Parts Out of Wholes

Setting: Various Rectangles

What to say: Share a candy bar among four friends. Cut off one share. What fraction of the whole strip did you cut off?



Disembedding: Fractions

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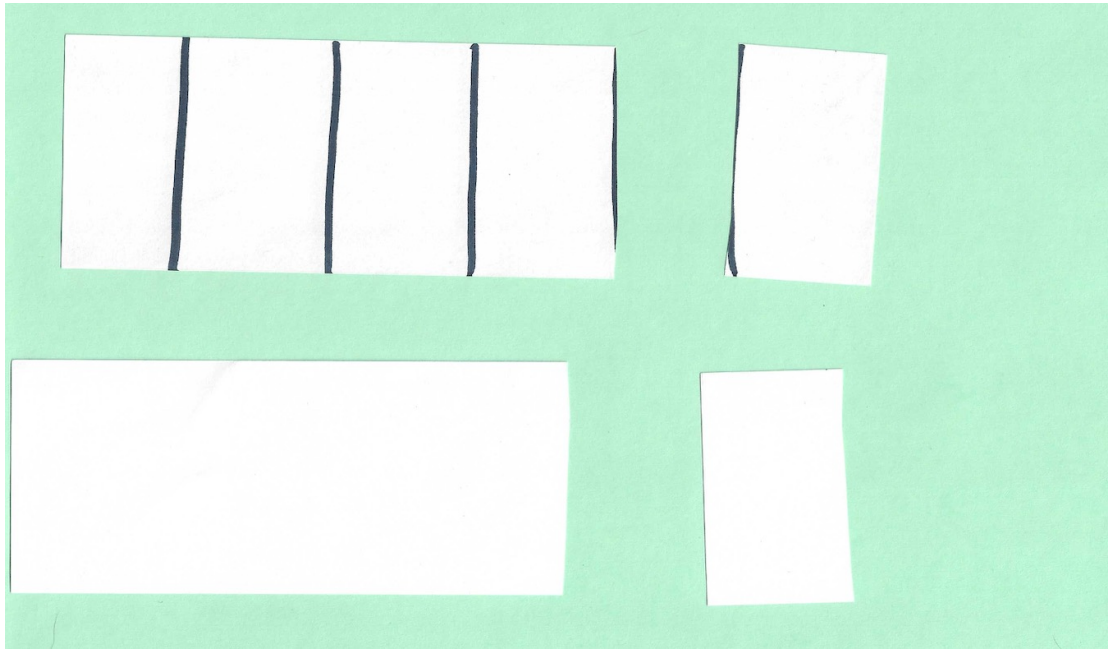


Operation (Mental Action)

- Taking a part out of a whole without destroying the whole

How do you support the mental action of disembedding?

Questioning

- What is the fraction name of the fraction you cut off?
- What is the fraction of the whole is left?

Task Selection

- Partitioned rectangles v. non-partitioned
- Creating one share v. Creating all shares
- Cut two or three shares from the whole

Mental Action: Iterating

- Producing connected copies of a quantity

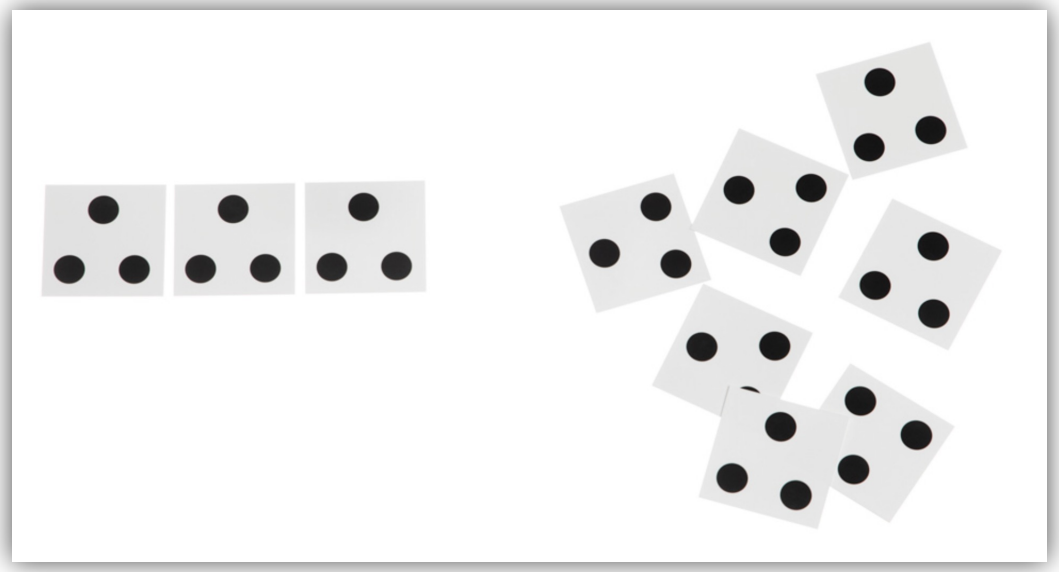
Iterating: Whole Number

Domain: Multiplication and Division

Topic: Early Multiplication (Counting-Based)

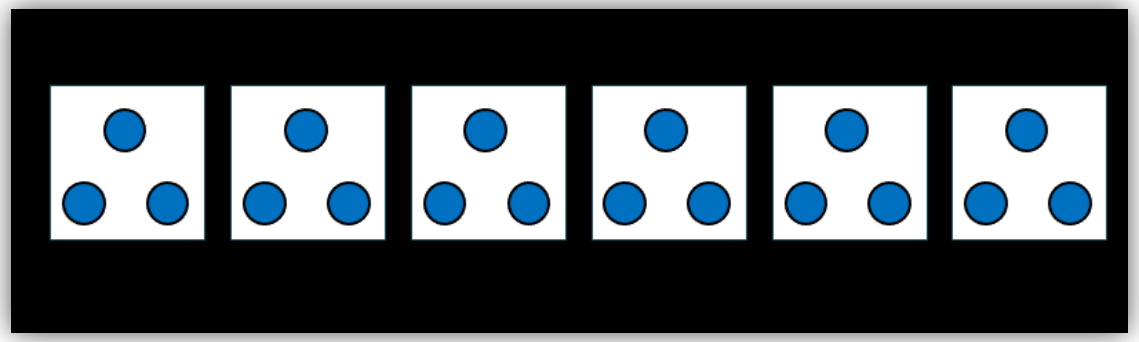
Setting: N-Groups

What to say: "There are 6 cards. There are 3 dots on each card. How many dots in all?"



Iterating: Whole Number

"There are 6 cards. There are 3 dots on each card. How many dots in all?"

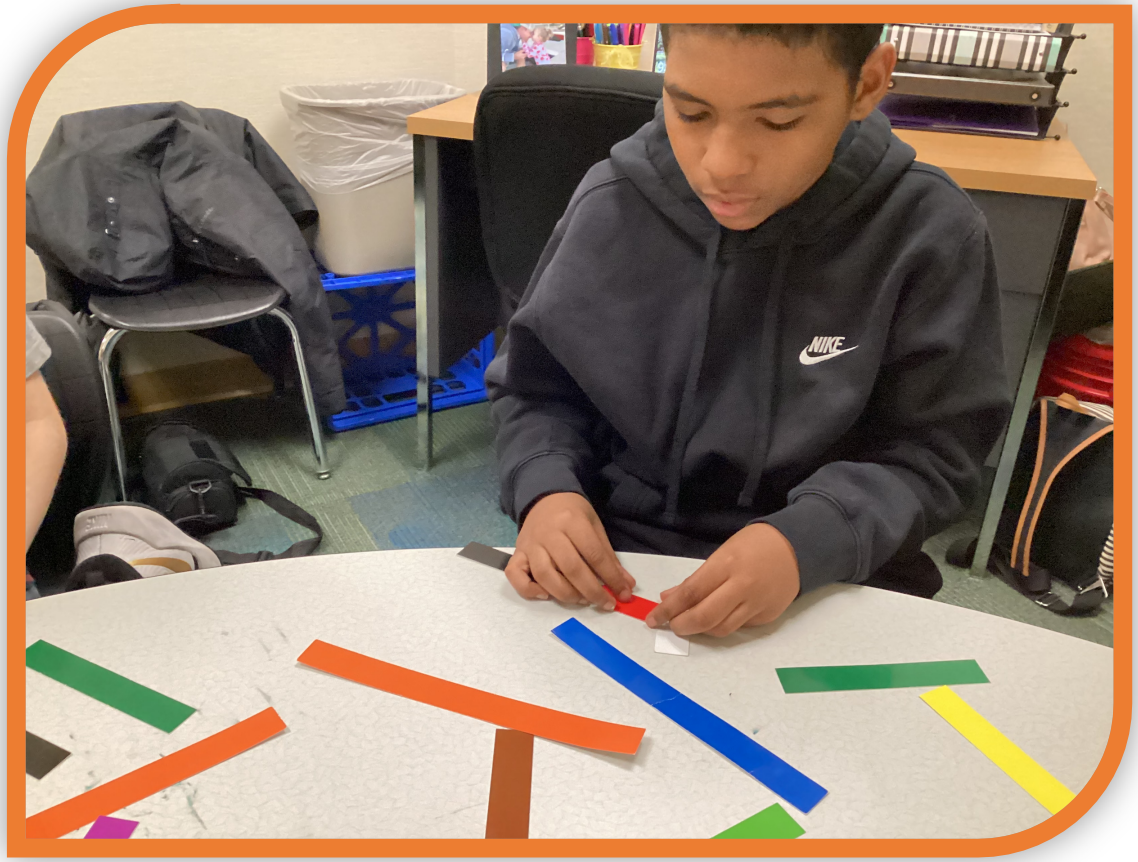


Operation (Mental Action)

- Students iterate composite units of 3 to solve

Iterating: Fractions

- Knowing fractions as quantities being produced from connected copies of a unit fraction
- Unit fraction becomes known to having a multiplicative relationship with a corresponding whole



Iterating: Fractions

Domain: Fractions

Topic: Fractions as Measures

Setting: Colored Fraction Strips

What to say: The red strip is 1/3. Which strip is the whole?

Iterating: Fractions

The red strip is 1/3. Which strip is the whole?

Operation (Mental Action)

- Producing equally sized parts or groups
- Repeating a 1/3 part three times to make a whole

How do you support the mental action of iterating?

Questioning

- What are you thinking about when trying to determine which strip is the whole?
- How do you know the strip you selected is the whole?
- Can you use the ___ strip to prove the strip you selected is the whole?

Task Selection

- Number of Iterations
- Colored Fractions Strips v. Drawing the whole or non-unit fraction

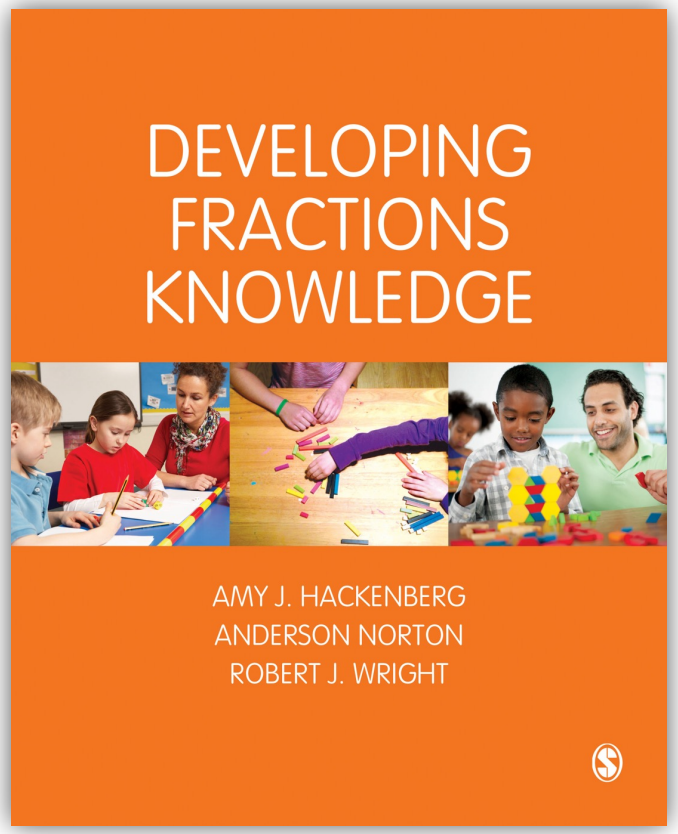
Implications of Mental Actions on Fractions Instruction

3.NF.A.3: Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

Fractions on the Number Line

- How do the mental actions support placing numbers on a number line?
- How can we scaffold the number to support students with the mental action of iterating?

Thank you!



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